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Emre Ozdenoren

Kathy Yuan

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JEL Classification: D86, G01, G30

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Emre Ozdenoren, London Business School and CEPR
Kathy Yuan, Department of Finance, Financial Markets Group, and Systemic Risk Centre, London School of Economics and Political Science

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Endogenous Contractual Externalities

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Abstract

We study effort and risk-taking behaviour in an economy with a continuum of principal-agent pairs where each agent exerts costly hidden effort. When the industry productivity is uncertain, agents have motivations to match the industry average effort, which results in contractual externalities. Contractual externalities have welfare changing effects when the information friction is correlated and the industry risk is not revealed. This is because principals do not internalise the impact of their choice on other principals' endogenous industry risk exposure. Relative to the second best, if the expected productivity is high, risk-averse principals over-incentivise their own agents, triggering a rat race in effort exertion, resulting in over-investment in effort and excessive exposure to industry risks relative to the second best. The opposite occurs when the expected productivity is low.

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*Ozdenoren is from the London Business School and CEPR; e-mail: `eozdenoren@london.edu`. Yuan is from the London School of Economics and CEPR; e-mail: `K.Yuan@lse.ac.uk`. We thank Ulf Axelson, Patrick Bolton, Peter Kondor, Thomas Mariotti, Ailsa Röell, Oguzhan Ozbas, Vania Stavrakev, Gunter Strobl, and seminar participants at the Frankfurt School of Finance and Economics, HKUST, Hong Kong University, Imperial College, London School of Economics, London Business School, Shanghai Advance Institute of Finance (SAIF), European Finance Association Meeting 2013, and European Winter Finance Meeting 2014 for helpful comments, and Oleg Rubanov for excellent research assistance. Yuan would like to acknowledge funding support from ERC and ESRC.

1 Introduction

It is important to understand the sources of industry boom and bust cycles, especially in light of the recent episodes in the high-tech and the finance industries. In these situations, the excited anticipation of the arrival of a "new technological era" of high productivities leads to over-investment and excessive risk-taking in the corresponding industry. This "overheating" in economic activities often contrasts with a subsequent crash where real investments and risks are substantially reduced. These pro-cyclical investment and risk-taking behaviours have significant social and economic consequences (eg, the recent great recession).

In this paper, we study a new mechanism based on frictions in contracting to explain pro-cyclical and potentially excessive risk-taking in the economy. In addition, our model contributes to the contract theory literature by endogenising systemic risk creation within a multiple principal-agent framework and provides a building block for studying excessive risk-taking behaviour in the economy. In our model, there are many firms in an industry. Each firm has a principal who owns a project and an agent who exerts costly hidden effort.[1] The return to effort of all agents is affected by an industry productivity shock. As a result, the level of industry risk faced by a firm is endogenous and is increasing in its agent's effort choice.[2] Additionally, the project's payoff is subject to idiosyncratic risk. Principals choose contracts to make risk-return trade-offs that are individually optimal. However, they do not take into account their impact on aggregate variables such as average effort in the industry. This results in contractual externalities when these aggregate variables enter the contracting problems as benchmarks. By investigating the conditions under which contractual externalities have welfare changing effects, our paper offers a new perspective on excessive risk-taking phenomenon over the boom-bust cycle.

In our baseline model, each principal uses a contract based on both absolute and relative performance evaluations (hereinafter APE and RPE). By using industry average as benchmarks in RPE, principals are able to shield their agents from industry risk which is correlated across agents but not from idiosyncratic risk. Hence, if they have to rely exclusively on RPE, principals encourage their agents to take on industry risk which they have to shoulder entirely. When it is feasible, principals would combine APE with RPE to improve risk sharing but the optimal weight on APE might be positive or negative. When principals care mostly

[1] This contrasts with setups with one principal and many agents (eg, team incentives) or many principals and one agent (eg, common agency).

[2] In this paper we treat the correlated industry risk and the systemic risk as the same and use the terms interchangeably.

about industry risk, they put positive weight on APE to expose the agent to industry risk and control 'excessive' industry risk taking. When principals care mostly about idiosyncratic risk, they put negative weight on APE to reduce agents' idiosyncratic risk exposure.[3]

These results from our baseline model offer a potential resolution for conflicting findings in the literature on RPE. Earlier empirical work has found that executives' compensations are very sensitive to industry performance.[4] These findings are interpreted as indirect evidence that little RPE is observed in practice, hence challenge the existing theory (Holmström 1979; 1982) which views the industry shocks as exogenous and unrelated to effort choices, and predicts that RPE will be used to make executives' compensations insensitive to such shocks. However, this interpretation conflicts with more recent empirical studies using a new source of data. Based on detailed disclosure data on executive compensation contracts, these studies find that a significant proportion of firms use some form of RPE.[5] Our results shed light on these seemly conflicting findings since in our theory the sensitivity to industry risk can be desirable which can be achieved by using a combination of RPE and APE. Furthermore, our model makes new testable predictions on how structural economic variables affect the optimal mix of RPE and APE.

We are also able to shed light on the related empirical observation that sensitivity of CEO compensation to industry shocks is asymmetric. CEOs are rewarded for good industry shocks but not punished for bad ones.[6] Literature has so far highlighted rent-seeking by CEOs as an explanation for these findings. We provide an alternative explanation based on optimal contracting. In our model, when there is a good industry shock and the expected industry productivity is high, agents put more effort resulting in more industry risks. Hence, principals would put positive weight on APE to control industry risk taking, seemingly rewarding the agent for industry performance. By contrast, when the expected industry productivity is low, idiosyncratic risks become relatively more important. This results in lower weight on AP and makes agents' compensations less sensitive to industry risks during downturns. In fact, our model makes the additional testable prediction that observed pro-cyclical pay sensitivity would be more pronounced in industries where industry shocks are

[3]Negative weight on APE can be surprising since it reduces incentive for effort provision. However, this can be optimal when it is very costly to let the agents take on additional idiosyncratic risk (eg., when agents are quite risk averse relative to principals or idiosyncratic shocks are very volatile). Moreover, negative weight on APE does *not* mean that agents are punished for good performance. When APE and RPE are *combined*, agents are always rewarded for their own performance.

[4]See Gibbons and Murphy (1990); Prendergast (1999); Aggarwal and Samwick (1999).

[5]See De Angelis and Grinstein (2010) and Gong et al. (2010).

[6]See Bertrand and Mullainathan (2001) and Garvey and Milbourn (2006).

large and principals are more risk-averse.

Next, we study how individually optimal contracting affects aggregate investment and risk-taking behaviour and its welfare implications using the baseline model as a building block. A key feature of our model is that the industry benchmark is endogenously determined because it is a function of the average managerial effort, an equilibrium outcome. As agents have incentives to match the industry benchmark to reduce their exposure to the industry productivity shock, this generates a feedback loop between individual and the industry average effort among agents. This feedback loop creates an externality in setting incentives among principals in the industry since principals take the industry benchmark as given and do not take into account the impact of their choices on it. To study the welfare impact of this externality we compare it with the second best where a planner maximises the sum of the payoffs of all the principals in the industry. When a principal gives stronger incentives to her agent, this leads to an increase in the industry average effort which has two effects. First, through the feedback loop, efforts of other agents, and hence expected outputs of all other firms increase. Second, higher efforts by other agents generate additional industry risks which are shouldered by all other principals. Since individual principals do not internalise their impact on other principals, relative to the second best, the first effect leads to too little while the second effect leads to too much effort provision.

We find that, in the baseline setup, where principals can use both APE and RPE to separate their agents' exposure to industry and idiosyncratic risks, contractual externalities do not have welfare impact, ie., the equilibrium outcome and the planner's solution coincide. However, we find that contractual externalities have welfare changing implications when there are informational frictions restricting the principals' ability to separate the two types of risks. The arrival of a new technological innovation, a phenomenon that often triggers industry boom and bust cycles is one important case. In reality absolute performances of CEOs are often measured by their firms' individual stock prices and the industry benchmark corresponds to the industry stock index. The hype around the new technology often causes a run up in all stock prices in the industry, without revealing the underlying industry productivity, eg, the dot.com boom in 1990s. The hype washes out when comparing individual firms' stock prices with the industry stock index. Hence, this type of information friction does not affect RPE but makes APE a noisy contractual instrument. As a result, principals rely more heavily on RPE less on APE, creating welfare changing effects of contractual externalities.

If the expected industry productivity is high, eg, during a boom, a principal in order to reap the high productivity benefit would like to elicit high effort from her agent by increasing incentives. Since APE is noisy, she relies more on RPE relative to the second best, which triggers a rat race among agents to exert effort and causes excessive industry risk exposure for principals. By contrast, if the expected industry productivity is low, eg, during a recession, the principal would like to reduce her agent's effort. Once again since APE is noisy, she reduces RPE instead. Relative to the second best, this triggers a race to the bottom to exert effort and generates too little industry risk. In this case, the planner can improve the total welfare by making RPE countercyclical: enforcing lower (higher) RPE during booms (busts). The model, therefore, offers some empirical predictions and policy guidance on managerial pay. For example, it predicts that social inefficiency is more likely in an industry where the industry-wide productivity is expected to be high and volatile and APE is noisy. This is more likely to be true in emerging industries as opposed to mature ones where there is less uncertainty about the industry productivity. Hence, it is relatively more important to have close supervision of excessive risk-taking in emerging industries with high expected productivities.

The mechanism described above leads to inefficiency in systemic but not in idiosyncratic risk taking. We show this by letting productivity shocks to be independent across projects. In this case, agents do not have motivation to match the industry average effort since they cannot remove their exposure to idiosyncratic productivity shocks in their compensations by matching their peers. Hence there is no feedback loop between individual and industry average effort. This unique prediction on inefficient procyclical systemic risk taking is well supported by the data. For example, Hoberg and Phillips (2010) find that boom-bust cycles are more likely in industries with many firms and when the common industry productivity shocks are volatile and difficult to predict. Bhattacharyya and Purnananda (2011) have documented between 2000 and 2006, the period of financial industry boom, idiosyncratic risks have dropped by almost half while systemic risks have doubled among US commercial banks. That is, the potentially excessive risk-taking during the boom period is found to be correlated among firms in the same industry.

The structure of the paper is as follows. In section 2, we discuss the related literature. In section 3, we present the model. In section 4, we lay out agents', principals' and planner's optimization problems. In section 5, we study the baseline case without any information frictions. We solve for the optimal linear contract under the equilibrium and the second best and compare the two. In sections 6 and 7, we study the case with information frictions, and analyse the welfare impact of contractual externalities. Section 8 concludes.

2 Related Literature

Since the results in our paper hinge on the fact that contracts put some weight on the industry average, our paper is closely related to the literature on relative performance (starting with Holmström 1979; 1982). We contribute to this literature theoretically in several aspects. First we endogenize the relative benchmark by linking it with equilibrium outcomes. Second, we study contractual externalities among multiple principal-agent pairs and their welfare consequences. Our theoretical extension has many unique predictions on the use of APE and RPE in compensation contracts that match well with the data as mentioned earlier and our comparative statics produce many new testable implications.

Many contracting situations involve rivalrous agency where principals hire agents who compete on their behalf. The literature on rivalrous agency (for example, Myerson (1982), Vickers (1985), Freshtman and Judd (1986; 1987), Sklivas (1987), Katz (1991), among others)) has examined the strategic aspect of incentive provision among principals to explore implications for oligopolistic conducts. Our paper differs in two aspects. First, in our model principals do not engage in direct competition and the interactions among agents arises endogenously via contracts that are based on correlated information. Second, our focus is different. We explore implications of contractual externalities for aggregate inefficiencies.[7]

Our model also relates to the principal-agent literature on effort and risk choice. In our model an agent's effort choice and the riskiness of his project are tightly linked. This is because the productivity of effort is random and correlated across firms, and thus when an agent increases his effort, both the expected return and the systematic risk exposure of the project are higher. We view this feature of the model desirable when studying excessive risk taking from a social perspective, especially considering that episodes of over (under) investment at the industry and/or the economy level are often observed together with excessive (insufficient) risk taking. In this way our setup and conclusions are different from those models where agents can choose effort and level of risk separately (See Diamond (1998); Biais and Casamatta (1999); Palomino and Prat (2003); and Makarov and Plantin (2010)). We acknowledge that in some settings agents can choose risk and return of the projects separately, however in other settings agents have to choose a portfolio of risk and

[7]Our model differs from the study of 'common agency' (Pauly (1974); Bernheim and Whinston (1986)) where multiple principals share the same agency since our principals do not share any agents. Furthermore, this literature studies contracting when externality is given. In our model, externality arises endogenously through contracting. Our paper is also different from the contract theory literature on (rank order) tournaments (Akerlof (1976); Lazear and Rosen (1981); Green and Stokey (1983); Nalebuff and Stiglitz (1983); and Bhattacharya and Mookherjee (1986)) where they study one principal and many agents.

return together. Put differently, agents may have to trade off investing effort in high-risk-high-return projects versus low-risk-low-return projects. By treating the risk-return as a portfolio, our framework complements and contributes to the understanding of sub-optimal risk-taking in the principal-agent framework. Importantly, we find that agents in our framework take suboptimal amounts of systematic rather than idiosyncratic risks since they can offset systematic risk exposures by matching the industry average effort. Additionally, in our framework sub-optimal risk taking arises due to contractual externalities as opposed to nonlinearities in payoff schedules.

The recent crisis has ignited an interest in macro and banking literature on excessive risk taking behaviour of banks. To our best knowledge, only one other line of literature predicts excessive undertaking of systemic risks and the prediction is one-sided about booms. This literature studies the incentive for banks to take on excessive risk collectively anticipating bailouts in case of financial crisis (Acharya and Yorulmazer (2007); Acharya and Yorulmazer (2008); Farhi and Tirole (2011); and Acharya et al. (2011)).

There is a line of financial literature that shows career or reputational concerns can lead to herd like behavior among agents (eg., Scharfstein and Stein (1990); Rajan (1994); Zwiebel (1995); and Guerrieri and Kondor (2012)). For example, Rajan (1994) models the information externality across two banks where reputational concerns and short-termism induce banks to continue to lend to negative NPV projects. He derives a theory of expansionary (or liberal) and contractionary (or tight) bank credit policies which influence, and are influenced by other banks' credit policies and conditions of borrowers. However, his model does not examine whether banks correlate their lending to similar industries or not. Further, in his model the short-term nature of managerial decisions drives career concern and hence expansionary bank credit policies during the boom, whereas in our model it is the information frictions on systemic productivity shocks. More broadly, the major difference between our paper and this line of research is that we study explicit rather than implicit incentives. This allows us to generate quite different and unique testable predictions and policy implications; eg, regulations on executive compensation over the business cycles.

3 Model

In this section, we describe our setup, information environment, and equilibrium definition.

3.1 The Setup

There is a continuum of principals in an industry. Each principal owns a firm which in turn owns a project. There is also a continuum of agents who are able to obtain a fixed reservation utility in a competitive labor market. The principal hires an agent to work on the project and offers the agent a contract. Each principal, agent and project triplet is indexed by $i \in [0, 1]$. The principal's objective is to maximize her expected utility which is based on the expected final value of the project. Principals are potentially risk averse, and their utility is given by $u_s(w) = -\exp(-r_s w)$ where $r_s \geq 0$.[8]

There are three dates $t = 0, 1, 2$. At $t = 0$, principal i offers agent $i \in [0, 1]$ a contract. We assume that contracts are offered simultaneously. Agent i observes his contract and decides whether to accept or reject it. If he accepts the contract, he chooses hidden effort denoted by e_i on project i. Agent i's effort is costly and the cost is specified as $C(e_i) = e_i^2/2$. We assume that all agents have identical CARA preferences so that $u(w_i, e_i) = -\exp(-r(w - C(e_i)))$ where $r \geq 0$. At $t = 1$, two payoff-relevant public signals about project i are revealed. One is about agent i's performance and the other is about the average performance of all projects in the industry. We assume that these signals are contractible and determine agent i's compensation. All agents are paid at time 1. At $t = 2$, the final values of all projects are realized and principals receive their payoffs. For simplicity we assume no discounting.

3.2 Production Technology

We assume that project i generates output V_i, which is a random function of agent i's unobservable effort and two stochastic shocks,

$$V_i = V(e_i, \tilde{h}, \tilde{\epsilon}_i). \tag{1}$$

The randomness arises from a common random variable $\tilde{h}$, and a project-specific random variable $\tilde{\epsilon}_i$. We interpret $\tilde{h}$ as a common productivity shock to all projects and $\tilde{\epsilon}_i$ as an output shock specific to the individual project. In the rest of the paper, we refer to $\tilde{h}$ as the industry productivity shock or the systemic shock as it cannot be diversified away. The

[8]Note that we allow for risk-averse principals. In presence of contractual externalities risk-neutrality of principals is not an innocuous assumption. Later in the paper, we discuss differences in the results when $r_s = 0$ and $r_s \neq 0$. In reality, there are a number of reasons why principals might be risk-averse or act as if they are risk-averse. Banal-Estanõl and Ottavinani (2006) have discussed these in detail, which include concentrated ownership, limited hedging, managerial control, limited debt capacity and liquidity constraints, and stochastic productions.

important assumption is that $\partial^2 V_i/(\partial h \partial e_i) \neq 0$, ie, the state of nature that is common across agents, affects the productivity of effort. This specification is meant to capture the uncertainty about industry productivity after a technological innovation.

Our results are based on a linear specification where $V_i = \tilde{h}e_i + \tilde{\epsilon}_i$. In our model, the random variable $\tilde{h}$ is normally distributed with mean $\bar{h} > 0$ and variance σ_h^2 (ie, precision $\tau_h = 1/\sigma_h^2$). The random variable $\tilde{\epsilon}_i$ is normally distributed with mean zero and variance σ_ϵ^2 (ie, precision $\tau_\epsilon = 1/\sigma_\epsilon^2$).[9]

Note that in our specifications, the productivity shock enters multiplicatively with effort. When $\sigma_h = 0$, the specification for output in our model is standard. In the more general case where $\sigma_h > 0$, higher average effort generates a higher return, but since the productivity of effort is random it also leads to higher volatility. Here, we have in mind a broad interpretation of effort as choosing the scale of the project, eg, by devoting more resources (time, personnel, etc.) to it.[10]

3.3 Information Structure

In our model principals receive contractible signals about the output of their individual projects, and the average output of the industry. We assume that the industry average reveals the industry productivity shock $\tilde{h}$ with noise. The idea is that after a major technological innovation there is uncertainty about industry productivity and it is difficult to assess the realisation of this uncertainty through public signals such as industry stock price indices, which themselves are very noisy.

Specifically, the first contractible signal is a noisy signal of project i's outcome, ie, agent i's performance, given by

$$s_i = \tilde{h}e_i + \tilde{\epsilon}_i + \tilde{\zeta}, \tag{2}$$

where $\tilde{\zeta}$ is an industry-wide noise normally distributed with mean zero and variance σ_ζ^2 (ie, precision $\tau_\zeta = 1/\sigma_\zeta^2$).

[9]To show that the common productivity shock is a key driver for our results, in Section 6.3, we analyse an alternative specification where the productivity function is $V_i = \tilde{k}_i e_i + \tilde{\epsilon}_i$. Here $\tilde{k}_i$ is a project-specific random term.

[10]Similar multiplicative function forms of productivity shocks and firm input choices have also been used to study firm dynamics with microeconomic rigidities in the macro literature. For example, Bloom et al. (2014) model the firm output as a triple multiplicative product of industry, idiosyncratic productivity shocks as well as firm's choices on capital and labor.

The second is a noisy signal of the industry average project outcome, ie, the average performance of all agents, given by

$$t = \tilde{h}\overline{e} + \tilde{\zeta}, \tag{3}$$

where $\overline{e} = \int_0^1 e_i di$ is the average effort of all agents. Note that since the signals about the projects' outcomes are correlated, the industry average output is observed with noise $\tilde{\zeta}$. Hence, the industry average reveals the industry productivity $\tilde{h}$ with noise.

In this paper, we restrict attention to linear compensation contracts which is common in the theoretical literature on principal-agent models. We let p_i be a signal about the agent's performance relative to his peers given by,

$$p_i = s_i - t = \tilde{h}\left(e_i - \overline{e}\right) + \tilde{\epsilon}_i. \tag{4}$$

In a linear contracting environment any contract written on s_i and t can be written in terms of s_i and p_i, and vice versa. To provide better intuition, in the rest of the paper, we assume that the principals write contracts on the relative performance signals rather than the industry average signal.

3.4 Equilibrium Definition

We assume that agent i's linear compensation contract has three components. The first component is a fixed wage W_i and the other two components condition the agent's payment on the realization of the two signals. Therefore, agent i's total compensation I_i is given by

$$I_i\left(l_i, m_i, W_i\right) = l_i p_i + m_i s_i + W_i, \tag{5}$$

where l_i and m_i are the weights on relative and absolute performance signals. Hence, l_i and m_i measure the relative performance evaluation (RPE) and absolute performance evaluation (APE) components of the contract.

Now we are ready to specify agent i's optimization problem. We assume that agents' reservation utility is $u\left(\bar{I}\right)$. Agent i accepts contract (l_i, m_i, W_i) if his expected utility from accepting the contract exceeds his reservation utility

$$E\left[u\left(I_i\left(l_i, m_i, W_i\right) - C\left(e_i\left(l_i, m_i, W_i\right)\right)\right)\right] = E\left[u\left(l_i p_i + m_i s_i + W_i - C\left(e_i\left(l_i, m_i, W_i\right)\right)\right)\right] \geq u\left(\bar{I}\right),$$

where $e_i\left(l_i, m_i, W_i\right)$ is the optimal effort choice conditional on accepting the contract. That is,

$$e_i\left(l_i, m_i, W_i\right) = \arg\max_{e_i \geq 0} E\left[u\left(l_i p_i + m_i s_i + W_i - C\left(e_i\right)\right)\right]. \tag{6}$$

We define an equilibrium of the model as follows.

Definition 1: An equilibrium consists of contracts (l_i^*, m_i^*, W_i^*), effort choices $e_i^* = e_i(l_i^*, m_i^*, W_i^*)$ for each $i \in [0,1]$ and average effort $\bar{e} = \int_0^1 e_i^* di$ such that given $\bar{e}$, the contract (l_i^*, m_i^*, W_i^*) solves principal i's problem, ie, it maximizes $E[u_s(V_i - I_i)]$ subject to $E[u(I_i - C(e_i))] \geq u(\bar{I})$, where $e_i = e_i(l_i, m_i, W_i)$ (given in (6)).

To study the potential externality in the economy, we also define the second best of the model. It is defined as the solution to the planner's problem where the planner maximises the sum of the utilities of all principals conditional on the incentive and individual rationality constraints for the agents. Formally,

Definition 2: A second-best solution consists of a contract (l^{SB}, m^{SB}, W^{SB}) and effort choice e^{SB} where $e^{SB} = e_i(l^{SB}, m^{SB}, W^{SB})$ and the contract solves the planner's problem, ie, it maximizes $\int_0^1 E[u_s(V_i - I_i)] di$, subject to $E[u(I_i - C(e_i))] \geq u(\bar{I})$, where $e_i = e_i(l, m, W)$ (given in (6)) .

Note that the planner's role is limited to coordinating the contracts written by principals. In particular, the planner must give agents incentives to accept the contract and exert the desired level of effort.[11]

We begin our analysis in section 4 by first solving the agents', principals' and planner's problems in the contractual environment discussed above. In section 5, we study a baseline case where $\tau_\zeta = \infty$ where there is no information friction regarding the uncertain industry productivity shock, $\tilde{h}$. In section 6, we incorporate in the model an information friction by letting $0 \leq \tau_\zeta < \infty$. In these cases, APE is not fully informative and principals rely more on RPE as contracting instruments. We discuss how the equilibrium effort level compares with the second-best and present results on comparative statics.

4 Agents', Principals' and Planner's Problem

In this section we first solve agents' equilibrium effort choices for a given contract. We then use this solution to characterize principals' and the planner's choices of optimal contract.

[11]The second best contract pushes agents exactly to their reservation utilities. However, it would be misleading to think that the second-best contract favours the principals' since given CARA utilities and linear contracts, the solution also maximizes the total surplus.

4.1 Agents' Effort Choice

Given contract (l_i, m_i, W_i) agent i's compensation is:

$$I_i = l_i p_i + m_i s_i + W_i = l_i \left(\tilde{h}\left(e_i - \bar{e}\right) + \tilde{\epsilon}_i\right) + m_i \left(\tilde{h} e_i + \tilde{\epsilon}_i \tilde{\zeta}\right) + W_i, \tag{7}$$

and agent i chooses e_i to maximize:

$$E\left[u\left(l_i\left(\tilde{h}\left(e_i - \bar{e}\right) + \tilde{\epsilon}_i\right) + m_i\left(\tilde{h} e_i + \tilde{\epsilon}_i + \tilde{\zeta}\right) + W_i - C\left(e_i\right)\right)\right]. \tag{8}$$

Computing the expectation in the above expression, agent i's problem in (8) can be restated as choosing e_i to maximize:

$$\left(l_i + m_i\right)\bar{h} e_i - l_i \bar{h}\bar{e} + W_i - C\left(e_i\right) - \frac{1}{2} r\left(\left(l_i\left(e_i - \bar{e}\right) + m_i e_i\right)^2 \frac{1}{\tau_h} + \left(l_i + m_i\right)^2 \frac{1}{\tau_\epsilon} + m_i^2 \frac{1}{\tau_\zeta}\right). \tag{9}$$

From (9) we see how a given incentive package shapes agent i's exposure to various sources of risks. His risk exposure to the common productivity shock ($\tilde{h}$) depends on (i) the power of the relative performance-based pay l_i times the difference between his effort and the average effort $(e_i - \bar{e})$, and (ii) the power of absolute performance pay m_i times his effort e_i. His risk exposure to the common noise $\tilde{\zeta}$ depends solely on the power of absolute performance-based pay while his risk exposure to the idiosyncratic noise $\tilde{\epsilon}_i$ depends on the power of *total* performance-based pay. From this we can see that by matching the average effort in the industry, agent i is able to completely hedge his exposure to the industry risk that comes through his relative performance pay, although he might still be exposed to some industry risk that comes through his absolute performance pay. Taking the first-order condition and solving for e_i, we obtain agent i's effort choice as

$$e_i = \frac{\left(l_i + m_i\right)\bar{h} + \frac{r}{\tau_h} l_i \left(l_i + m_i\right)\bar{e}}{1 + \frac{r}{\tau_h}\left(l_i + m_i\right)^2}. \tag{10}$$

Note that agent i's effort is increasing in $\bar{e}$, the average effort exerted by all the other agents with a positive relative performance pay sensitivity. Thus, when the average effort increases, agent i's best response is to increase his effort.

Typically, the more risk averse an agent is (ie., the higher r is) and/or the more volatile the industry shock becomes (ie., the lower τ_h is), the lower effort level he will choose. This is because by lowering his effort, the agent reduces his exposure to the industry risk. This effect is captured by the term r/τ_h in the denominator of (10). In our setup, the term r/τ_h is also in the numerator capturing the fact that when r is higher or τ_h is lower, an agent has a stronger incentive to match the average effort to hedge the industry risk. Through this second effect, for a given contract (l_i, m_i), the agent's effort may increase when his risk aversion is higher or the industry productivity shock becomes more volatile.

Crucially the total performance-based pay, $(l_i + m_i)$, has two opposing effects on agent i's effort choice. Increasing it makes agent i increase his effort because his pay becomes more sensitive to average productivity, $\bar{h}$, and the magnitude of his performance relative to the industry average $\bar{e}$. This is captured by the numerator in (10). At the same time, increasing $(l_i + m_i)$ causes agent i to bear more industry risk by making the agent deviate more from the industry average. This increase in risk exposure induces him to lower his effort. This effect is reflected by the denominator in (10). Moreover, both effects become stronger as the industry risk $1/\tau_h$ increases. As we show later, these two effects underly the externalities that principals have to face when writing the compensation contracts. Note that in the limit, as the industry risk approaches zero, our model delivers the standard result where agent i's effort is determined by his performance pay and the productivity of his effort, ie, $(l_i + m_i)\bar{h}$.

4.2 Principals' Choice of Optimal Contract

Now we turn to the principals' problem. Principal i chooses the contract terms (l_i, m_i, W_i) to maximize her expected utility, $E\left[u_s\left(V_i - I_i\right)\right]$ subject to $E\left[u\left(I_i - C\left(e_i\right)\right)\right] \geq u\left(\bar{I}\right)$ where e_i is given by (10).

We proceed to solve the equilibrium contract terms (l_i, m_i, W_i). Using (7) we obtain principal i's final payoff as

$$V_i - I_i = \tilde{h}e_i + \tilde{\epsilon}_i - l_i\left(\tilde{h}\left(e_i - \bar{e}\right) + \tilde{\epsilon}_i\right) - m_i\left(\tilde{h}e_i + \tilde{\epsilon}_i + \tilde{\zeta}\right) - W_i.$$

Computing $E\left[u_s\left(V_i - I_i\right)\right]$, we see that principal i chooses (l_i, m_i, W_i) to maximize

$$\left(1 - l_i - m_i\right)\bar{h}e_i + l_i\bar{h}\bar{e} - W_i - \frac{1}{2}r_s\left(\left(e_i - l_i\left(e_i - \bar{e}\right) - m_ie_i\right)^2\frac{1}{\tau_h} + \left(1 - l_i - m_i\right)^2\frac{1}{\tau_\epsilon} + m_i^2\frac{1}{\tau_\zeta}\right) \quad (11)$$

where e_i is given by (10). Using (9) and agent i's individual rationality constraint we obtain

$$-\left(l_i + m_i\right)\bar{h}e_i + l_i\bar{h}\bar{e} - W_i = -C\left(e_i\right) - \frac{1}{2}r\left(\left(\left(l_i + m_i\right)e_i - l_i\bar{e}\right)^2\frac{1}{\tau_h} + \left(l_i + m_i\right)^2\frac{1}{\tau_\epsilon} + m_i^2\frac{1}{\tau_\zeta}\right) - \bar{I}.$$

We substitute the above equation into (11) to see that principal i chooses (l_i, m_i) to maximize

$$\begin{aligned}&\bar{h}e_i - C\left(e_i\right) - \frac{1}{2}r_s\left(\left(e_i - l_i\left(e_i - \bar{e}\right) - m_ie_i\right)^2\frac{1}{\tau_h} + \left(1 - l_i - m_i\right)^2\frac{1}{\tau_\epsilon} + m_i^2\frac{1}{\tau_\zeta}\right)\\&-\frac{1}{2}r\left(\left(l_i\left(e_i - \bar{e}\right) + m_ie_i\right)^2\frac{1}{\tau_h} + \left(l_i + m_i\right)^2\frac{1}{\tau_\epsilon} + m_i^2\frac{1}{\tau_\zeta}\right) - \bar{I}.\end{aligned} \quad (12)$$

The above expression has an intuitive interpretation as it is principal i's and agent i's combined surplus. The first term is the expected output of the project, the second term is the cost of agent i's effort, and the next two terms are the disutilities from the risk exposures of the agent and the principal respectively.

From the above expression, we see that APE and RPE play different roles in risk sharing between principals and agents. APE introduces agents to both industry and idiosyncratic risks. By contrast, when agents match each other's effort choices, RPE shields agents from industry risk, although it still exposes agents to idiosyncratic risk.

In this paper we will restrict attention to situations where the equilibrium is unique. Next proposition guarantees the existence of a unique equilibrium as long as the industry risk is not too large.[12]

PROPOSITION 1: Given $\bar{h}$, r, r_s, there exists $\bar{\tau}_h$ such that for all $\tau_h > \bar{\tau}_h$ there exists a unique equilibrium contract which is symmetric.

Note that once the values of $\bar{h}$, r, r_s are fixed, Proposition 1 guarantees that there is a unique equilibrium for large enough τ_h regardless of the values of τ_ϵ and τ_ζ. [13]

4.3 Planner's Problem

From Definition 2 we see that the planner chooses the contract terms l and m to maximize the sum of principals' utilities subject to incentive and participation constraints. Since principals' optimization problems are identical, the planner's problem can be seen equivalently as maximizing the utility of one of the principals taking into account that $e_i^* = \bar{e}$. That is, the planner internalizes the impact of the contract terms on the industry average effort level $\bar{e}$. Thus, the planner chooses (l, m) to maximize

$$\begin{aligned}&\bar{h}e - C(e) - \frac{1}{2}r_s\left(e^2(1-m)^2\frac{1}{\tau_h} + (1-l-m)^2\frac{1}{\tau_\epsilon} + m^2\frac{1}{\tau_\zeta}\right)\\&-\frac{1}{2}r\left(m^2e^2\frac{1}{\tau_h} + (l+m)^2\frac{1}{\tau_\epsilon} + m^2\frac{1}{\tau_\zeta}\right) - \bar{I},\end{aligned} \tag{13}$$

where

$$e = \frac{(l+m)\bar{h}}{1 + \frac{r}{\tau_h}(l+m)m}. \tag{14}$$

[12]When the industry risk is large, it is possible to construct examples of multiple equilibria. The multiplicity of equilibrium contracts is an interesting possibility that is worth studying further in future work.

[13]This allows us to fix τ_h and perform comparative statics with respect to τ_ϵ and τ_ζ (without losing existence and uniqueness of the equilibrium).

In our model, industry benchmark is a function of the industry average effort. Since agents have incentives to match the industry benchmark to reduce their exposure to the industry productivity shock, this generates a feedback loop between individual and the industry average effort of the agents. By comparing equations (12) and (13), we observe that in the decentralised equilibrium principals do not internalise their choices of contract terms on the industry average effort while the planner does. As a result, this feedback loop creates externalities, which we term as contractual externalities, in the decentralised equilibrium where the principals do not take into account their impact on the industry benchmark. Comparing the decentralised equilibrium outcome with the second best allows us to investigate the magnitude and the direction of these contractual externalities and perform comparative statics.

5 The Baseline Model

In this section, we study the baseline set up where $\tau_\zeta \to \infty$ and the noise $\tilde{\zeta}$ disappears. As we show below, in this case information friction regarding the uncertain industry shock $\tilde{h}$ is absent. We begin our analysis by explicitly characterising the equilibrium in this baseline case.

Proposition 2: When τ_ζ approaches infinity, the optimal contract (l^*, m^*, W^*) is symmetric and unique. The total performance sensitivity $a^* = l^* + m^*$ is the unique positive root to the following equation:

$$\begin{aligned}
&\bar{h}^2 \left(\frac{r}{\tau_h} a + 1\right) \left(\frac{r}{\tau_h} + \frac{r_s}{\tau_h}\right)^2 (a-1) \\
&+ \left(\frac{r}{\tau_h} + \frac{r_s}{\tau_h} + \left(\frac{r}{\tau_h}\right)^2 a^2 \left(\frac{r_s}{\tau_h} + 1\right) + 2\frac{r_s}{\tau_h}\frac{r}{\tau_h} a\right)^2 \left(a\frac{r}{\tau_\epsilon} + (a-1)\frac{r_s}{\tau_\epsilon}\right) = 0.
\end{aligned} \tag{15}$$

Given a^* the contract term m^* is given by:

$$m^* = a^* - \frac{\frac{r}{\tau_h}\left(1 + \frac{r}{\tau_h}(a^*)^2\right) + \frac{r_s}{\tau_h}(a^* - 1)\left(\frac{r}{\tau_h}a^* + 1\right)}{\left(\frac{r}{\tau_h}a^* + 1\right)\left(\frac{r}{\tau_h} + \frac{r_s}{\tau_h}\right)}. \tag{16}$$

The equilibrium contract of the baseline model features both APE and RPE, although the optimal weight on APE, m^*, might be positive or negative. Corollary 2 characterizes the sign of APE in equilibrium.

Corollary 1: The weight on the absolute performance signal m^*, is positive (negative, zero) if

$$\bar{h}^2 \frac{r_s}{\tau_h}\left(\frac{r_s}{\tau_h}+1\right)+\frac{r_s}{\tau_h}\frac{r_s}{\tau_\epsilon}\left(1+\frac{r}{\tau_h}\right)+\frac{r}{\tau_\epsilon}\left(\frac{r_s}{\tau_h}-\frac{r}{\tau_h}\right) \tag{17}$$

is positive (negative, zero).

It is interesting to note that when m^* is strictly positive, agents are rewarded for better industry performance. In contrast, in the single-agent relative performance model, under corresponding assumptions, agents would not be rewarded by what seems to be luck rather than effort.[14] The difference in the results is rooted in the industry risk.

We can see from the principal's objective function in (12) that when the agents match the average effort in the industry, they do not face any industry risk through RPE. The only industry risk they face comes from APE. In this sense, like in Holmström (1982), RPE completely filters out the correlated risk or the luck component. At the same time, the fact that RPE shields them from the industry risk means that the agents do not consider the impact of their effort choice on their firm's exposure to the industry risk, potentially exposing their principals to it excessively. Therefore, different from the classical relative performance literature, our model finds that principals use APE to control and share risks with agents which RPE alone cannot achieve.

Specifically, APE plays two roles from risk-sharing perspective. First, by exposing agents to the industry risk, it reduces their incentive to take on industry risk. Second, it offsets agents' idiosyncratic risk exposure. The condition in Corollary 1 shows which of these forces prevails in equilibrium. For example, when principals are risk averse and average return to effort and/or industry risk is high, the optimal contracts puts a positive weight on APE (ie., $m > 0$) so that agents would internalise their tendency to take on too much industry risk. By contrast, when principals are close to risk-neutral and average return to effort and/or industry risk is low, the optimal contract puts a negative weight on APE (ie., $m < 0$) to reduce the agents' idiosyncratic risk exposure.

[14] In the standard relative performance model principal observes two signals: a noisy signal of the agent's performance and a second signal that is uninformative about the agent's performance but correlated with the noise term of the first signal. The second signal could be the performance of other agents working on the project but could also be any other information correlated with the signal about the agent's performance. When the two signals are positively correlated, the second signal gets a negative weight. This is because when the second signal is higher, the principal learns that the noise in the first signal is likely to be high. Putting a negative weight on the second signal, allows the principal not to reward the agent for luck.

This finding regarding the purposes of APE versus RPE in compensation contracts offers a unique explanation to various empirical puzzles. For example, the empirical phenomenon of "paying for luck" might be due to the fact that principals want to control agents' excessive risk-taking tendency. This empirical fact is established by running regression of executive pay on industry benchmarks. However, as we show, there might be a large amount of RPE in the compensation contracts (high l) even when the pay is positively correlated with industry risk (high m). This simple regression only reflects the net effect of APE and RPE and is no longer sufficient. Our model shows that principals' usage of APE and RPE is more complex in the presence of both industry and idiosyncratic risks and a careful decomposition of the pay package to undercover this underlaying cause of a particular mix of APE and RPE instruments is needed instead. Furthermore, based on Corollary 1, our model predicts the "pay for luck" phenomenon occurs more often in industries with volatile and high expected productivity, while in industries where expected productivity is low, and firm-specific risks are larger, our model finds that the sensitivity to industry risks is much lower, even turns negative, predicting an asymmetry in "paying for luck." These are new testable implications.

Next we turn to the comparison of the the decentralised equilibrium and the planner's solution in the baseline case.

PROPOSITION 3: When τ_ζ approaches infinity, the effort choices and contracts coincide in equilibrium and in the planner's solution.

In other words, if the industry productivity shock is perfectly revealed, principals are able to completely counteract the impact of externalities among agents' effort-taking through optimal contracting. To see this algebraically, let τ_ζ go to infinity, set $a_i = l_i + m_i$ and $c_i = l_i \bar{e}$. Substituting these in (12) we can restate principal i's problem as choosing (a_i, c_i) to maximize:

$$\bar{h}e_i - \frac{1}{2}r_s\left((e_i - a_i e_i + c_i)^2 \frac{1}{\tau_h} + (1-a_i)^2 \frac{1}{\tau_\epsilon}\right) - C(e_i) - \frac{1}{2}r\left((a_i e_i - c_i)^2 \frac{1}{\tau_h} + a_i^2 \frac{1}{\tau_\epsilon}\right) - \bar{I}$$

where agent i's effort is given by

$$e_i = \frac{a_i \bar{h} + \frac{r}{\tau_h} a_i c_i}{1 + \frac{r}{\tau_h} a_i^2}$$

Note that stated this way principals' problems are completely separated and $\bar{e}$ no longer plays a role. This is because principal i can completely eliminate the impact of the industry average effort $\bar{e}$ by adjusting c_i. By redefining the principals' optimization problem this way, we see that it coincides with the planner's problem and Proposition 3 is obvious.

Intuitively, when information friction on industry risk is absent, principals can use the two contractual instruments – APE and RPE – to fine tune their agents' exposures to the two types of risks – industry and idiosyncratic – and undo the welfare effect of the contractual externality regardless of the industry average effort. The planner, therefore, has no role to play in this environment. Here we observe a parallel between the workings of contractual and pecuniary externalities. In general, pecuniary externality also does not have welfare changing effects except for conditions as established in Stiglitz (1982), Greenwald and Stiglitz (1986), Geanakoplos and Polemarchakis (1985), Arnott, Greenwald and Stiglitz (1994), and more recently Farhi and Werning (2013).[15]

In the following two sections, we extend the baseline model to $0 \leq \tau_\zeta < \infty$. In these cases principals receive noisy and correlated signals about absolute performances, and have to rely more on relative performance information. We illustrate how the resulting information friction restricts the principals' ability to separate the two types of risks, shapes the contracts and generates welfare changing effects.

6 Information Friction

Often major technological innovations make it extremely difficult to assess the productivity of an industry but it is still possible to evaluate an agent's performance relative to his peers. To capture this feature in the simplest way, we begin our analysis by allowing the industry-wide noise on APE to be extremely volatile, that is, by letting τ_ζ be zero.[16]

[15]There is an explosion of the literature on the welfare effect of pecuniary externalities due to the growing interests in studying social inefficiency of booms-busts. This includes but not limited to the following: Krishnamurthy (2003); Caballero and Krishnamurthy (2001; 2003); Gromb and Vayanos (2002); Korinek (2010); Bianchi (2010); Bianchi and Mendoza, (2011); Stein (2012); Gersbach and Rochet (2012); He and Kondor (2013); Farhi and Werning (2013). Davila (2011) and Stavrakeva (2013) have nice summaries of this literature. Similar to pecuniary externalities, we show later that, with frictions, contractual externalities might have welfare changing effects.

[16]Intuitively, when there is a great uncertainty about the industry productivity, it is relatively easy to assess an agent's performance relative to his peers. That is, the information on the ranking of agents is more precise than the information on an agent's absolute performance level. Empirically, we observe that stock analysts are better at ranking stocks than pricing stocks (Da and Schaumburg (2011)). The finance literature is more successful in explaining cross-sectional equity returns while the equity premium remains a puzzle. Moreover, this information structure parsimoniously captures the tournament-like incentives that agents face in the real world. For example, the ranking of businesses, university programs, fund managers, doctors in different specialities, and even economists of different vintages, is prevalent when there is also (possibly quite noisy) information on their individual performance.

In this limiting case, principals do not have any information about $\tilde{h}$ directly, and both signals s_i and t are uninformative by themselves. However, their difference p_i is informative because it is unaffected by the common noise $\tilde{\zeta}$. Consequently, principals can only assess how much better or worse their agents are performing relative to the average and have to base agents' compensation on this information alone. As a result, $m_i^* = 0$, that is, contracts do not include an absolute performance-based pay component. In section 7, we relax this assumption and study the intermediate case of $0 < \tau_\zeta < \infty$.

To solve her problem, principal i takes $\bar{e}$ as given and chooses the optimal linear contract which we denote by l_i^*. The following proposition characterizes the equilibrium contract and effort levels.

Proposition 4: When $\tau_\zeta = 0$, under the conditions in Proposition 1, a unique symmetric equilibrium contract exists and satisfies

$$\frac{\bar{h}^2}{\frac{r}{\tau_h}\left(l^*\right)^2 + 1}\left(1 - l^*\right)\left(1 - \frac{r_s}{\tau_h} l^*\right) - \frac{1}{\tau_\epsilon}\left(r l^* - r_s\left(1 - l^*\right)\right) = 0. \tag{18}$$

Moreover, $l^* \in (0, 1)$ and the equilibrium effort level is $e^* = \bar{e} = l^* \bar{h}$.

6.1 Equilibrium Properties

The expositional clarity of the equilibrium RPE (l^*) in (18) allows us to explore further properties of contracts in this economy. To illustrate, we dissect the equilibrium condition (18) into terms that reflect the tradeoff between incentives and risk-sharing. To do so we define the *incentive provision* as the level of compensation when the sole purpose of the contract is to incentivise the agents to exert effort, and the *risk-sharing provision* as the level of compensation when the purpose of the contract is to allow risk sharing between principals and agents. The following corollary characterizes the optimal contract in two limiting cases.

Corollary 2: When τ_ϵ goes to infinity, the optimal linear contract reflects only the incentive provision and is given by $l_i^* = \min\{1, \tau_h / r_s\}$. When τ_ϵ goes to zero, the optimal linear contract reflects only the risk-sharing provision and is given by $l_i^* = r_s/(r_s + r)$.

Corollary 2 allows us to identify the terms in the equilibrium condition (18) that correspond to incentive and risk sharing provisions:

$$\bar{h}^2 \underbrace{\frac{1}{\frac{r}{\tau_h}\left(l_i^*\right)^2+1}}_{\substack{\text{Cost of Unilateral Deviations}\\ \text{in Incentive Provision}}} \underbrace{\left(1-l_i^*\right)\left(1-\frac{r_s}{\tau_h} l_i^*\right)}_{\text{Incentive Provision}} - \frac{1}{\tau_\epsilon} \underbrace{\left(r l_i^* - r_s\left(1-l_i^*\right)\right)}_{\text{Risk-Sharing Provision}} = 0. \tag{19}$$

The magnitude of risk-sharing provision is standard and depends on the relative risk-aversions of principals and agents, $r_s/(r_s+r)$. The magnitude of the incentive provision has aspects unique to our model. In the standard moral hazard framework the magnitude of incentive provision is simply 1. This is because when there is no risk sharing concern, it is optimal to "sell the project" to the agent. A key insight of our model is that this intuition does not hold when there is endogenous risk creation by the agents and this risk is borne disproportionately by the principals. In fact, in our model, principals shoulder all industry risk in equilibrium and the amount of industry risk depends on agents' effort choices.[17] Principals take account of the endogenous industry risk and their appetite for it and set $l_i^* = min\{1, \tau_h/r_s\}$ when there is no risk sharing concern. Thus, the magnitude of incentive provision is less than 1 when industry productivity is volatile or principals are risk averse enough.

The weights that the incentive and the risk-sharing concerns receive in the equilibrium contract are given by their coefficients in (19). The ratio of these two coefficents captures the relative importance of the two concerns.

The decomposition in (19) shows that industry and idiosyncratic risks affect the relative importance of incentive provision through different channels. Because idiosyncratic risks are shared, when $1/\tau_\epsilon$ goes up, the importance of incentive provision relative to risk sharing declines. The impact of industry risk is more subtle. It affects the relative importance of incentive provision through the term $(r(l_i^*)^2/\tau_h + 1)$.[18] This term is affected by agent i's risk aversion and captures his disutility from taking on additional industry risk when incentivised to work (potentially) more than the industry average.[19] Note that this cost is not incurred by agents in equilibrium. Nevertheless it plays a role in the determination of the equilibrium contract. This is because a principal, considering unilateral deviation from

[17]To see why this is this case, recall in equilibrium $e_i^* = \bar{e}$. This implies that each agent's industry risk exposure in his compensation contract is zero in equilibrium (from (9)).

[18]This term appears in (10) when we solve agent i's optimal effort (except that here $m = 0$).

[19]Of course, in equilibrium, agent i's industry risk exposure in his compensation contract is zero since agent i hedges industry risk by choosing $e_i^* = \bar{e}$. Since each principal takes other principals and agents behaviours as given, in her view, deviating from equilibrium choice and providing stronger incentives unilaterally would impose her agent to bear more risk and hence result in this additional cost of incentive provision.

equilibrium, would take this cost into account.

Next we highlight comparative statics that are unique to our model with potentially new empirical implications.[20] In the standard moral hazard framework, the power of contracts increases in the marginal productivity of effort $\bar{h}$ and the precision of idiosyncratic risk τ_ϵ. As the next proposition shows, in our model, this is not necessarily the case.

Proposition 5: If $\tau_h/r_s < (>, =)\, r_s/(r_s + r)$, l^* decreases (increases, is constant) in $\bar{h}$ and τ_ϵ.

To understand this proposition first note that the importance of incentive provision relative to risk sharing is increasing in $\bar{h}$ and τ_ϵ. In the standard moral hazard framework, the magnitude of incentive provision is 1 and it always exceeds the magnitude of risk sharing provision $r_s/(r_s + r)$. Hence, when the relative importance of incentive provision increases, the power of the contract also increases. In contrast, in our model, as we explained above due to endogenous risk taking, it is possible to have the magnitude of incentive provision smaller than that of risk sharing provision. In this case, when the relative importance of incentive provision increases, the power of the contract decreases.

Comparative statics of the equilibrium contract l^* with respect to the principals' and the agents' risk aversion parameters, r_s and r, also provide new empirical implications. In the standard moral hazard setting, as the principal becomes more or the agent becomes less risk averse, l^* increases to provide better risk-sharing. The next two propositions illustrate that in the present setting there are opposing effects which can dominate the direct effect of improved risk sharing.

Proposition 6: If $\bar{h}$ or τ_ϵ are large enough, l^* decreases in r_s.

Since the principal needs to shoulder the entire industry risk, as she becomes more risk averse, importance of incentive provision goes down as the importance of risk sharing goes up. Proposition 6 shows that when $\bar{h}$ or τ_ϵ are large, this effect dominates the direct effect, and l^* decreases in r_s.

Proposition 7: If $\tau_h/r_s < r_s/(r_s + r)$ and $\bar{h}$ or τ_ϵ are large enough, l^* increases in r.

[20] To test these implications, it is possible to obtain empirical proxies for the model parameters such as industry (marginal) productivity, industry risks and idiosyncratic risks, as well as risk aversions of the principals and agents. For example, one can use the proportion of institutional investors in the shareholder base of a firm as a proxy for (the inverse of) risk aversion of the firm.

The intuition for Proposition 7 is more subtle. Suppose magnitude of risk sharing provision is larger than incentive provision, i.e., $\tau_h/r_s < r_s/(r_s+r)$. As r increases, the importance of risk-sharing relative to incentive provision goes up, and the power of the contract increases. Proposition 7 shows that when $\bar{h}$ or τ_ϵ are large, this effect dominates the direct effect and l^* increases in r.

6.2 Comparison with the Second Best

Next, for the case $\tau_\zeta = 0$, we compare the equilibrium efforts and contracts with their second-best levels. Recall that second-best solves the problem of the planner who internalizes the impact of the contracts on the industry average effort level $\bar{e}$. As we discussed in Section 4.3, the planner's problem can be viewed as maximizing the objective function given in (13) subject to agents' effort choices given in (14). Since, when $\tau_\zeta = 0$, the planner optimally sets $m = 0$, from (14) we obtain $e = l\bar{h}$. Plugging this into (13), the planner's problem becomes

$$\max_{l \geq 0} \left[-\frac{1}{2}\bar{h}^2 l \left(l \left(1 + \left(\frac{r_s}{\tau_h} \right) \right) - 2 \right) - \frac{1}{2} l^2 \frac{r}{\tau_\epsilon} - \frac{1}{2} (1-l)^2 \frac{r_s}{\tau_\epsilon} \right]. \tag{20}$$

The first-order condition of the problem is

$$\underbrace{\bar{h}^2 \left(1 - l^{SB} \left(\frac{r_s}{\tau_h} + 1 \right) \right)}_{\text{Incentive Provision}} - \frac{1}{\tau_\epsilon} \underbrace{\left(r l^{SB} - r_s \left(1 - l^{SB} \right) \right)}_{\text{Risk-Sharing Provision}} = 0, \tag{21}$$

and the solution to the planner's problem is:

$$l^{SB} = \frac{\frac{r_s}{\tau_\epsilon} + \bar{h}^2}{\frac{r}{\tau_\epsilon} + \frac{r_s}{\tau_\epsilon} + \bar{h}^2 \left(\frac{r_s}{\tau_h} + 1 \right)}. \tag{22}$$

Like the optimal equilibrium contract, the second-best solution also reflects the incentive and risk-sharing provisions. The following corollary summarizes the limiting results for the second-best contract.

Corollary 3: When τ_ϵ goes to infinity, the second-best contract reflects only incentive provision and is given by $l^{SB} = 1/(r_s/\tau_h + 1)$. When τ_ϵ goes to zero, the second-best contract reflects only risk-sharing provision and is given by $l^{SB} = r_s/(r_s + r)$.

Although the second-best solution of (21) is similar to the equilibrium solution of (19) in reflecting both incentive and risk-sharing provisions, there are two important differences. First, the second best requires a lower magnitude of incentive provision than in equilibrium.[21]

[21] Since $1/(r_s/\tau_h + 1) < min\{1, \tau_h/r_s\}$.

Second, there is no cost of unilateral deviations in incentive provision. That is, contractual externality has two opposing effects. Intuitively, the first effect arises because principals do not take into account the industry risk exposure of other principals in the industry. When principals are risk averse, they have to trade off incentivising their agents to work harder versus exposing themselves to more industry risks in their projects. Stronger the incentive they choose, higher the output they would expect, and larger the industry risk they are exposed. Their industry risk exposure is endogenously linked to the strength of the incentives they provide. When setting incentives, a principal optimally chooses her own risk-return tradeoff ignoring her impact on increasing other principals' industry risk exposure. In the second best, a planner sets incentives by taking into account the feedback loop between industry average and individual effort choices and consequences of industry risk exposure for other principals in the industry. This means, the second best requires weaker incentives for agents.

The second effect goes in the opposite direction and arises because each principal perceives a unilateral deviation from the industry average as being too costly. Recall, the cost of unilateral deviations in incentive provision is incurred in equilibrium when a principal, who takes the industry average effort $\bar{e}$ as given, considers increasing incentives and making her agent work harder unilaterally. The principal realizes that by doing so, her agent's effort would be above $\bar{e}$ which imposes costly industry risk on the agent, and she has to compensate the agent for this risk. In the second best, this unilateral deviation cost disappears because planner can coordinate (dictate) incentive provision across all principals in the industry. Therefore, the relative importance of incentive provision is higher in second best.

To summarise, the externality in the model has two opposing effects on the performance-pay sensitivity in the contract. Compared with the second best, *the magnitude* of equilibrium incentive provision is larger because principals do not internalise the impact of their incentive provision on the average effort level and the industry risk exposure of other principals, consequently, provide too much incentive. However, *the relative importance* of equilibrium incentive provision is lower because principals perceive unilateral increases in incentive provision as too costly. The next proposition characterises which effect dominates and whether the equilibrium contract is more or less sensitive to performance than the second-best contract.

PROPOSITION 8: The equilibrium contract is more (less, equally) sensitive to performance than the second-best contract (ie, l^{SB} is smaller than (greater than, equal to) l^*), and consequently agents put more (less, equal) effort in equilibrium than the second best, (ie, e^{SB} is less than (greater than, equal to) e^*) if (17) is positive (negative, zero).

Comparing Proposition 8 and Corollary 1 we immediately obtain the following result linking the usage and the sign of APE in the baseline model with the direction of inefficiencies that result from basing contracts on RPE alone.

COROLLARY 4: When $\tau_\zeta = 0$ and contracts are based solely on RPE, the equilibrium contract is more (less, equally) sensitive to performance than the second-best contract if and only if without any informational friction (ie., when $\tau_\zeta = \infty$), the weight on the absolute performance signal m^*, is positive (negative, zero).

Corollary 4 gives a different perspective on the results of excessive/insufficient risk taking with only RPE. As we discussed in Section 5, principals would like to use positive APE, ie, set $m > 0$, in order to incentivise their agents while letting them internalise the industry risk they are generating. However, when principals are constrained from using APE, they end up relying more on RPE, ie, set a larger l, triggering feedback loops between the industry average and agents' effort choices, causing excessive effort provision and risk taking in equilibrium relative to the second best. Similarly, when principals are constrained from using negative APE to control effort, the opposite happens. They lower RPE instead, triggering a race to the bottom, resulting in insufficient equilibrium effort and risk-taking relative to the second best.

The above proposition shows a pro-cyclical pattern of incentive provision, effort choice and risk-taking in the economy. To see this, note that (17) is positive for a sufficiently large $\bar{h}$ if principals are risk-averse. When $\bar{h}$ is large, the incentive provision term gets a larger weight in equilibrium than in the second-best (shown as the coefficient in front of the incentive concern term in equations (19) and (21)). This means that when $\bar{h}$ is large, ie, during the productivity boom, the contracting between principals and agents is more motivated by the incentive concern. During this time, the expected productivity of effort is very high, and principals would like to offer their own agents a contract with a high performance sensitivity. By doing so, they do not internalise the impact of their own incentive-provision on increasing the industry average effort, and trigger a rat race. Since marginal productivity of effort is random in our model, an immediate consequence of this result is that there is excess risk-taking behaviour among agents in equilibrium. The planner, in this case, can improve the total welfare by enforcing lower performance-based pay sensitivities in agents' compensation contracts.

By contrast, when $\bar{h}$ is low, eg., during downturns, (17) is negative. In this case, since the expected productivity of effort is low, the incentive provision term gets a lower weight in equilibrium than in the second-best. The cost of providing incentives unilaterally becomes a major consideration for principals. Principals would like to free-ride on each other in incentive provision, offering their agents a contract with a low performance-pay sensitivity. By doing so, principals again do not internalise the impact of their own incentive-provision on increasing the industry average effort, and hence under-incentivise the agents relative to the second-best. This again triggers a race but this time causes a race to the bottom. There is insufficient effort- and risk-taking. In this case, the planner can improve the total welfare by enforcing contracts with higher performance based pay-sensitivities.

6.3 Industry-wide vs. Idiosyncratic Variations in Productivity

In this section, we show that the excessive (insufficient) effort provision is related to the common/systemic rather than project-specific/idiosyncratic risk. To highlight the source of externality we consider the case where the productivity shock is idiosyncratic rather than common to the industry. Specifically, we let $V_i = \tilde{k}_i e_i + \tilde{\epsilon}_i$ where $\tilde{k}_i$ is a project-specific random term which is independently and normally distributed across agents with mean $\overline{k}$ and variance $1/\tau_k$.

As before, we assume that the two contractible signals are

$$s_i = \tilde{k}_i e_i + \tilde{\epsilon}_i + \tilde{\zeta},$$

and

$$t = \overline{k}\bar{e} + \tilde{\zeta},$$

where $\bar{e}$ is the average effort exerted by the agents in the industry. Clearly this is equivalent to simply observing $p_i = \tilde{k}_i e_i - \overline{k}\bar{e} + \tilde{\epsilon}_i$.

We can now write agent i's compensation when absolute performance signals are not contractible (eg., $\tau_\zeta = 0$) as

$$I_i = l_i p_i + W_i = l_i \left(\tilde{k}_i e_i - \overline{k}\bar{e} + \tilde{\epsilon}_i \right) + W_i. \tag{23}$$

Using (23), given a contract (l_i, W_i) and average effort $\bar{e}$, agent i chooses e_i to maximize

$$E\left(u\left(I_i - C\left(e_i\right)\right)\right).$$

Plugging in p_i and computing the expectation in the above equation, the agent's problem can be restated as choosing e_i to maximize

$$l_i \overline{k} e_i - l_i \overline{k}\bar{e} + W_i - C\left(e_i\right) - \frac{1}{2} r \left(\left(l_i e_i\right)^2 \frac{1}{\tau_k} + l_i^2 \frac{1}{\tau_\epsilon} \right).$$

Taking the first-order condition and solving for e_i, we obtain agent i's effort choice as

$$e_i = \frac{l_i \bar{k}}{1 + \frac{r}{\tau_k} l_i^2}. \tag{24}$$

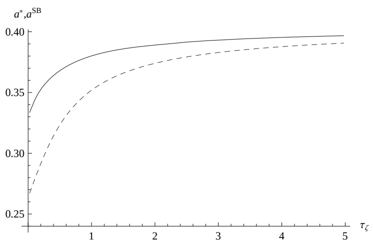
The above equation shows that, as one would expect, a volatile project-specific risk ($1/\tau_k$) lowers the effort level. More importantly, it shows that when the productivity shock is idiosyncratic, there are no feedback loop between the industry average effort and an individual agent's effort. Therefore, the results we obtained earlier on excessive (insufficient) effort provision can only arise in an environment where the productivity shock has a systematic component across projects in the industry.

7 Intermediate Cases of Information Friction

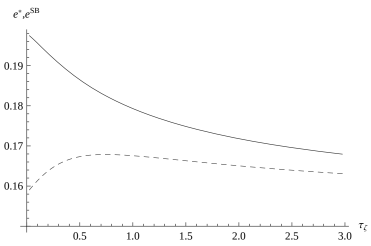
In the previous two sections, we derive closed-form solutions and explore the properties of the model with either no information frictions or with severe information frictions when only RPE is informative. These cases correspond to τ_ζ equal to infinity or zero. In this section, we look at the intermediate cases where $0 < \tau_\zeta < \infty$, that is, principals receive an informative but imperfect signal about absolute performances.

In these cases, the information friction does not eliminate, but nevertheless restricts principals' ability to use APE in contracts, causing externalities to prevail. Since, a closed form solution is not possible, we provide two numerical examples, showing how the equilibrium and the second best incentives change with information friction, τ_ζ. One example is a case when contractual externalities cause excessive effort/risk taking relative to the second best; and the other is the opposite. In both examples, when τ_ζ increases, the impact of the endogenous contractual externality becomes smaller as principals' ability to span the risk space of the agents strengthens. Therefore, our numerical analysis indicates that when the noise $\tilde{\zeta}$ becomes more precise, the impact of the externality weakens and the gap between the equilibrium and the second best narrows.

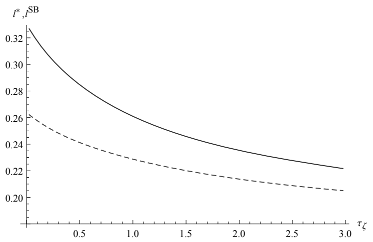
The graphs in Figure 1 illustrate the intuition in the case where equilibrium effort level exceeds the second best. In this case, (17) is positive indicating that, without information friction, principals would like to use positive APE ($m^* > 0$). As this noise becomes smaller (ie., τ_ζ gets larger), the information friction on using APE is less constraining, principals increase the equilibrium sensitivity to APE (m^*) to give agents a positive exposure to the industry risk and better control agents' excessive correlated risk-taking. This is shown in Figure 1(d). Correspondingly, this switch to the usage of APE in contracts leads to the



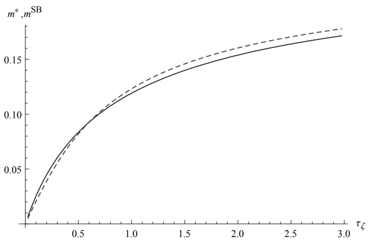
(a) a^* and a^{SB}



(b) e^* and e^{SB}

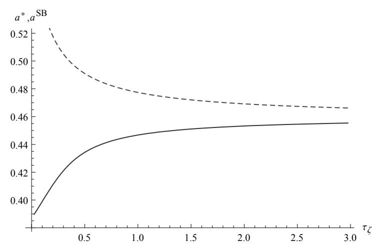


(c) l^* and l^{SB}

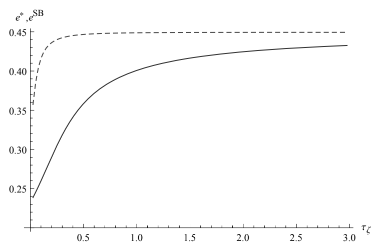


(d) m^* and m^{SB}

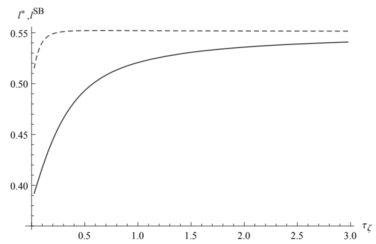
Figure 1: Noisy Industry Signal (τ_ζ) and Excessive Effort: The solid and the long-dashed lines represent how the total performance sensitivity a, relative performance l, absolute performance m, and effort (e) change with respect to the noise of the average industry performance signal (τ_ζ) in equilibrium and in the planner's optimum, respectively. The parameters are fixed at $r = 0.3$, $r_s = 0.16$, $\tau_\epsilon = 1$, $\bar{h} = 0.6$, and $\tau_h = 0.05$.



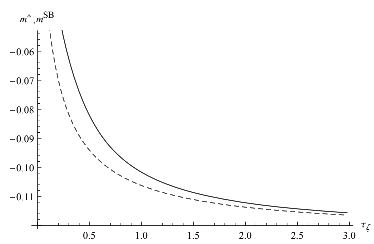
(a) a^* and a^{SB}



(b) e^* and e^{SB}



(c) l^* and l^{SB}



(d) m^* and m^{SB}

Figure 2: Noisy Industry Signal (τ_ζ) and Insufficient Effort: The solid and the long-dashed lines represent how the total performance sensitivity l, relative performance l, absolute performance m, and effort (e) change with respect to the noise of the average industry performance signal (τ_ζ) in equilibrium and in the planner's optimum, respectively. The parameters are fixed at $r = 0.3$, $r_s = 0.01$, $\tau_\epsilon = 1$, $\bar{h} = 0.6$, and $\tau_h = 0.05$.

sensitivity to relative performance (l^*) to drop in equilibrium, as shown in Figure 1(c). However, the total performance sensitivity in the equilibrium contract ($a^* = l^* + m^*$) increases since principals are able to use both contractual instruments, APE and RPE, more effectively as τ_ζ increases. Interestingly, agents reduce effort in equilibrium because they are now exposed to the industry risk through absolute performance pay, as shown in Figure 1(b). Further, as expected, when τ_ζ gets larger, the information friction gets smaller, the impact of the externality weakens, the gap between the equilibrium and the second best narrows.

The graphs in Figure 2 illustrate the intuition in the case where equilibrium effort level falls below the second best. In this case, (17) is negative indicating that, without information friction, principals would like to use negative APE ($m^* < 0$). This happens when principals are less risk-averse and/or idiosyncratic risks are relatively larger than the industry risks. In these occasions, principals would not mind taking over a large portion of idiosyncratic risks from their agents to lower contracting costs, which implies negative absolute performance pay sensitivity. The noise in the industry output signal, $\tilde{\zeta}$, constrains principals' ability to do so. When τ_ζ gets larger, this constraint gets less binding, which explains the increased use of APE (that is, the contract is more negatively related to absolute performance) in Figure 2(d). This in turns makes it less costly for principals to use RPE since agents bear less firm-specific risks, resulting in higher usage of relative performance pay as in Figure 2(c). The total performance sensitivity in the equilibrium contract (a^*), just as in the previous case, also increases in τ_ζ. Again when the noise in absolute performance signals becomes smaller, principals are less constrained to use both contractual instruments, APE and RPE, to span the risk space agents are facing, as shown in Figure 2(a). Interestingly, agents increase their effort in equilibrium because they are less exposed to the idiosyncratic risk through the lowered absolute performance sensitivity and more incentivized to take on correlated industry risk through increased relative performance sensitivity, which is shown in Figure 2(b). Further, as expected, when τ_ζ gets larger, the information friction gets smaller, the impact of the externality weakens, the gap between the equilibrium and the second best narrows. These numerical examples show that the insights of the models are robust and endogenous contractual externalities exist when there exists some form of the information friction in the contracting environment.

8 Conclusion

In this paper, we study how information frictions in contracting leads to pro-cyclical and potentially excessive risk-taking in the economy. In our model, principals set contracts to make individually optimal risk-return trade-offs ignoring their impact on contracting benchmarks such as average effort in the industry. This results in contractual externalities. In our baseline model, absolute performance signals do not have any correlated noise and contracts are based on both APE and RPE. We show that by shielding agents from it, RPE encourages agents to take industry risk which the principals must shoulder. Despite this, principals use the two contractual instruments to tailor their agents' exposures to the industry and idiosyncratic risks and eliminate the welfare impact of contractual externalities.

However, in presence of information frictions contractual externalities have welfare changing effects. For example, when there is a high level of uncertainty about industry productivity, relative performance information is likely to be more precise and principals lean on RPE in contracting. Overreliance on RPE may set off the ratchet effect in effort choices among agents. For example, risk-averse principals are eager to provide more powerful incentives during booms, causing the industry average effort to be high, triggering a rat race among agents to exert excessive effort, which results in excessive systemic risk exposure in the economy, relative to the second-best. During recessions, the opposite might happen: The incentive provision is too weak and the equilibrium level of effort is lower than the second best.

Besides theoretical contributions, our results offer a novel explanation, based on frictions in managerial compensation, to the boom-bust cycle of investment and risk-taking observed in industries that experience new but uncertain productivity shocks. These episodes are abundant in recent years. For example, following the introduction of the Internet, the dot.com industry has been flooded with investment which is subsequently reduced. Similarly, following the innovations in the financial products such as asset-based securities, the financial industry has expanded significantly followed by a sharp contraction. Compensation regulations such as enforcing counter-cyclical performance pay could improve the total welfare.

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Appendix

In the proof we drop the subscript i when there is no room for confusion and we use the following notation:

$$t = \frac{r}{\tau_h},\ v = \frac{r}{\tau_\epsilon},\ u = \frac{r}{\tau_\zeta},\ p = \frac{r_s}{\tau_h},\ q = \frac{r_s}{\tau_\epsilon},\ s = \frac{r_s}{\tau_\zeta}.$$

Proof of Proposition 1

Note that

$$e_i = \frac{(l_i + m_i)\,\bar{h} + t l_i (l_i + m_i)\,\bar{e}}{1 + t (l_i + m_i)^2} = (l_i + m_i)\,\bar{h} + o\left(1/\tau_h\right).$$

Let $a_i = l_i + m_i$. Substituting this into (12) we obtain:

$$\max_{a_i, b_i} a_i \left(\bar{h}\right)^2 - \frac{\left(a_i \bar{h}\right)^2}{2} - \frac{1}{2}\left(a_i^2 v + m_i^2 u\right) - \frac{1}{2}\left((1 - a_i)^2 q + m_i^2 s\right) + o\left(1/\tau_h\right) - \bar{I}.$$

For sufficiently large τ_h this function is concave in (a_i, m_i) and has a unique maximum (a^*, m^*) which is identical for all principals. From this we solve for the unique $l^* = a^* - m^*$.

Proof of Proposition 2:

Let $a = l + m$. The principals' objective can be written as

$$\bar{h}e - \frac{1}{2}e^2 - \frac{1}{2}\left((ae - l\bar{e})^2 t + a^2 v\right) - \frac{1}{2}\left(((1 - a)\,e + l\bar{e})^2 p + (1 - a)^2 q\right). \tag{25}$$

and the first order condition yields the optimal level of effort as a function of contract terms and the average effort

$$e = \frac{a\bar{h} + tla\bar{e}}{1 + ta^2}.$$

Next substituting for the effort level in the objective function of (25) we obtain

$$\left(\bar{h}\left(\frac{a\bar{h} + tla\bar{e}}{1 + ta^2}\right) - \frac{1}{2}\left(\frac{a\bar{h} + tla\bar{e}}{1 + ta^2}\right)^2 - \frac{1}{2}\left(\left(a\left(\frac{a\bar{h} + tla\bar{e}}{1 + ta^2}\right) - l\bar{e}\right)^2 t + a^2 v\right)\right) \tag{26}$$
$$-\frac{1}{2}\left(\left((1 - a)\left(\frac{a\bar{h} + tla\bar{e}}{1 + ta^2}\right) + l\bar{e}\right)^2 p + (1 - a)^2 q\right).$$

For a given a the above function is negative quadratic in l. Thus for a given a principals' objective function is maximized at $l(a)$ which is given by

$$l(a) = \frac{\bar{h}a\left(t\left(ta^2 + 1\right) - p(1 - a)(ta + 1)\right)}{\bar{e}\left(t\left(ta^2 + 1\right) + p(ta + 1)^2\right)}. \tag{27}$$

Substituting (27) for $l(a)$ in (26) we reduce principals' problem to choosing a to maximize

$$\frac{1}{2}\bar{h}^2 a(t+p)\frac{a(t-1)+2}{t+p+t^2a^2+pt^2a^2+2pta}-\frac{1}{2}a^2v-\frac{1}{2}(1-a)^2q.$$

Taking the derivative with respect to a, we obtain

$$\frac{-\bar{h}^2(ta+1)(t+p)^2(a-1)}{(t+p+t^2a^2+pt^2a^2+2pta)^2}-av-(a-1)q.$$

Note that the above function starts as positive and crosses to negative once. Thus the objective function is maximized at a^* that solves

$$H(a)=-\bar{h}^2(ta+1)(t+p)^2(a-1)-\left(t+p+t^2a^2(p+1)+2pta\right)^2(av+(a-1)q)=0.$$

Note that $a^*\in(0,1)$. In equilibrium

$$\bar{e}=\frac{a^*\bar{h}+ta^*l(a^*)\bar{e}}{1+t(a^*)^2}.$$

Plugging for $l(a^*)$, we obtain

$$\bar{e}=\frac{\bar{h}a^*(ta^*+1)(t+p)}{t\left(t(a^*)^2+1\right)+p(ta^*+1)^2}.$$

Using the above to substitute for $\bar{e}$ in (27), we obtain

$$l(a^*)=\frac{t\left(1+t(a^*)^2\right)+p(a^*-1)(ta^*+1)}{(ta^*+1)(t+p)}.$$

Thus

$$\begin{aligned}
m^* &= a^*-l(a^*)\\
&= a_i^*-\frac{\frac{r}{\tau_h}\left(1+\frac{r}{\tau_h}(a_i^*)^2\right)+\frac{r_s}{\tau_h}(a_i^*-1)\left(\frac{r}{\tau_h}a_i^*+1\right)}{\left(\frac{r}{\tau_h}a_i^*+1\right)\left(\frac{r}{\tau_h}+\frac{r_s}{\tau_h}\right)}.
\end{aligned}$$

Proof of Corollary 1:

We continue to use the notation in the proof of Proposition 2**.** First, note that

$$m^*=a^*-l(a^*)>0\Leftrightarrow l(a^*)<a^*.$$

Using the expression for x_{a^*}we see that

$$l(a^*)<a^*\Leftrightarrow p(1+ta^*)>t(1-a^*),$$

or if and only if

$$a^* > \left(\frac{t-p}{t\,(1+p)}\right).$$

Note

$$H\left(\frac{t-p}{t\,(1+p)}\right) = \frac{1}{t}\frac{1}{(p+1)^3}\,(t+p)^2\,(t+1)^2\left(pv - tv + \bar{h}^2 p^2 + \bar{h}^2 p + pq + pqt\right).$$

Thus $l\,(a^*) \leq a^*$ if and only if

$$\left(\bar{h}^2 \frac{r_s}{\tau_h}\left(\frac{r_s}{\tau_h}+1\right) + \frac{r_s}{\tau_\epsilon}\frac{r_s}{\tau_h}\left(\frac{r}{\tau_h}+1\right)\right) \geq \frac{r}{\tau_\epsilon}\left(\frac{r}{\tau_h} - \frac{r_s}{\tau_h}\right)$$

which is equivalent to (17).

Proof of Proposition 3

Proof is given in the text following the statement of Proposition 3.

Proof of Proposition 4

From Proposition 1, we know the existence part holds.

We use (10) to plug in for e in the principals' problem (12) (where m is set to zero given that the signal t is uninformative) and take the derivative of the objective function with respect to l to find the first-order condition as a function of $\bar{e}$.

In equilibrium, $\bar{e} = l\bar{h}$. Therefore any equilibrium must solve for the first-order condition and $\bar{e} = l\bar{h}$. To find an equilibrium we plug $\bar{e} = l\bar{h}$ in the first order condition. After simplifying we find the equilibrium condition:

$$\bar{h}^2 \frac{(l-1)\,(lp-1)}{tl^2+1} - (vl - q\,(1-l)) = 0.$$

The next lemma is useful in proving the comparative statics results:

Lemma 1: Let

$$\Psi\,(l) = \bar{h}^2 \frac{(l-1)\,(lp-1)}{tl^2+1}. \tag{28}$$

Suppose there is a unique equilibrium, then the following are true. (i) $\Psi\,(l) - (vl - q\,(1-l))$ crosses zero from above at l^* where Ψ is given in (28). (ii) If $1/p > q/\,(v+q)$ then $l^* < 1/p$, otherwise $l^* > 1/p$.

Proof of Lemma 1

Part (i) follows from $\Psi(0) + q > 0$, $\Psi(1) - v < 0$ and uniqueness. The proof of (ii) is immediate if $1/p > 1$. So suppose $1/p < 1$. Otherwise, $\Psi(l)$ crosses zero at $1/p < 1$. $\Psi(l) - (vl - q(1-l))$ is above $\Psi(l)$ for $l < q/(v+q)$ and is below $\Psi(l)$ for $l > q/(v+q)$. This and the fact that there is a unique equilibrium l^* prove part (ii).

Proof of Proposition 5

Equilibrium l^* solves

$$\Psi(l^*) - (vl^* - q(1-l^*)) = 0$$

where Ψ is given in (28). We write $\Psi\left(l^*\left(\bar{h}\right), \bar{h}\right)$ to make the dependence of Ψ and l^* on $\bar{h}$ explicit. We use similar notation for other parameters, eg, $\Psi(l^*(\tau_h), \tau_h)$.

Taking the total derivative of the equilibrium condition with respect to $\bar{h}$ we obtain

$$\frac{\partial l^*\left(\bar{h}\right)}{\partial \bar{h}} = \frac{-\frac{\partial \Psi}{\partial \bar{h}}}{\frac{\partial \Psi\left(l^*\left(\bar{h}\right), \bar{h}\right)}{\partial l} - (v+q)}.$$

Denominator is negative by Lemma 1 (i). By Lemma 1 (ii),

$$\frac{\partial \Psi\left(l^*, \bar{h}\right)}{\partial \bar{h}} = -2\bar{h}\frac{(1-l^*)(pl^*-1)}{t(l^*)^2+1} \gtreqless 0$$

if $1/p \gtreqless q/(v+q)$ which proves part (i) for $\bar{h}$. Proof for the result on τ_ϵ is entirely analogous.

Proof of Proposition 6

Taking the total derivative of the equilibrium condition with respect to r_s, we obtain

$$\frac{\partial l^*}{\partial r_s} = -\frac{\frac{\partial \Psi}{\partial r_s} + \frac{1}{\tau_\epsilon}(1-l^*)}{\frac{\partial \Psi}{\partial l} - (v+q)}$$

Denominator is negative by Lemma 1 (i). Moreover,

$$\frac{\partial \Psi}{\partial r_s} + \frac{1}{\tau_\epsilon}(1-l^*) = \bar{h}^2\frac{(l-1)\left(\frac{l}{\tau_h}\right)}{tl^2+1} + \frac{1}{\tau_\epsilon}(1-l^*)$$

Hence $\frac{\partial l^*}{\partial r_s} < 0$ if

$$\bar{h}^2\tau_\epsilon > \left(\frac{r}{\tau_h}(l^*)^2 + 1\right)\frac{\tau_h}{l^*} \tag{29}$$

On the r.h.s. of (29) only l^* depends on $\bar{h}$ or τ_ϵ. We know that l^* takes a value between $\frac{r_s}{r+r_s}$ and $\frac{\tau_h}{r_s}$. Hence the r.h.s. is bounded in $\bar{h}$ and τ_ϵ. As a result (29) holds if $\bar{h}$ or τ_ϵ are large.

Proof of Proposition 7

Taking the total derivative of the equilibrium condition with respect to r, we obtain:

$$\frac{\partial l^*(r)}{\partial r} = \frac{-\frac{\partial \Psi}{\partial r} + \frac{1}{\tau_\epsilon} l^*}{\frac{\partial \Psi}{\partial l} - (v+q)}.$$

Denominator is negative by Lemma 1 (i). Thus $\frac{\partial l^*(r)}{\partial r} > 0$ iff

$$\frac{\partial \Psi(l^*, r)}{\partial r} - \frac{1}{\tau_\epsilon} l^* = \bar{h}^2 \frac{(1-l^*)(pl^*-1)(l^*)^2}{\left(t(l^*)^2+1\right)^2} \frac{}{\tau_h} - \frac{1}{\tau_\epsilon} l^* > 0.$$

From Lemma 1 (ii), $1/p < q/(v+q)$ implies $l^* > 1/p$. Hence, the above inequality holds iff

$$\bar{h}^2 \tau_\epsilon > \frac{\left(\frac{r}{\tau_h}(l^*)^2+1\right)^2 \tau_h}{(1-l^*)\left(\frac{r_s}{\tau_h} l^* - 1\right) l^*}. \tag{30}$$

On the r.h.s. of (30) only l^* depends on $\bar{h}$ or τ_ϵ and $\frac{\tau_h}{r_s} \leq l^* \leq \frac{r_s}{r+r_s}$. Hence the r.h.s. is bounded in $\bar{h}$ and τ_ϵ. As a result (30) holds if $\bar{h}$ or τ_ϵ are large.

Proof of Proposition 8

Proof follows from plugging l^{SB} in the equilibrium condition (18) and checking whether its value is positive (in which case $l^{SB} < l^*$) or negative (in which case $l^{SB} > l^*$).

The London School of Economics
and Political Science
Houghton Street
London WC2A 2AE
United Kingdom

Tel: +44 (0)20 7405 7686
systemicrisk.ac.uk
src@lse.ac.uk