The Macroeconomic Impact of Microeconomic Shocks: Beyond Hulten's Theorem

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Macroeconomic Impact of Shocks

- For economy with efficient equilibrium, Hulten (1978):

$$\mathrm{d}\log C / \mathrm{d}\log A_i = sales_i / GDP = \lambda_i.$$

- First-order approximation or log-linearization.
- Foundation for Domar aggregation:
  - sales approximate sufficient statistics.
  - details of production structure are irrelevant.
- "Bugbear" for production networks literature.
  (shocks to Walmart and electricity equally important)

What We Do

- Extend Hulten to second order to capture nonlinearities.
- General formula: reduced-form macro-elasticities of substitution.
- Mapping from micro to macro using a general structural model:
  - structural micro elasticites of substitution.
  - returns to scale.
  - factor market reallocation.
  - network linkages.
- Nonlinearities lead to asymmetric responses of output to shocks.
  - amplification of negative shocks, attenuation of positive shocks.
  - lower mean, negative skewness, excess kurtosis.
- Nonlinearities quantitatively important:
  - $\times 10$ welfare costs of business cycles from 0.05% to 0.6% of GDP.
  - $\times 4$ impact of 70's oil price shocks from -0.7% to -2.4% of GDP.

Related Literature

- Long and Plosser (1983), Horvath (2000), Gomme and Rupert (2007).
- Jovanovic (1987), Durlauf (1993), Scheinkman and Woodford (1994), Horvath (1998), Dupor (1999).
- Gabaix (2011), Carvalho and Gabaix (2013), Acemoglu et al. (2012), Carvalho (2010), Acemoglu et al. (2017), Foerster et al. (2011), Atalay (2016), Bigio and La'O (2016), Baqaee (2016), Di Giovanni et al. (2014), Pasten et al. (2017).
- Kremer (1993), Jones (2011), Jones (2013).
- Houthakker (1955), Jones (2005), Oberfield and Raval (2014), Boehm and Oberfield (2017), Beraja et al. (2016).

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General Framework

- Perfectly competitive economy, representative consumer.
- Preferences represented by CRS consumption-bundle metric

$$C = \mathscr{C}\left(c_1, \ldots, c_N\right),$$

where c_i is consumption of good i.

- Consumer budget constraint

$$\sum_i p_i c_i = \sum_{i=1}^{M} w_i l_i + \sum_{i=1}^{N} \pi_i,$$

where p_i, w_i, and π_i are prices, wages, and profits.

General Framework

- Profits earned by the producer of good i:

$$\pi_i = p_i y_i - \sum_{k=1}^{M} w_k l_{ik} - \sum_{j=1}^{N} p_j x_{ij}.$$

- Each good i is produced using production function:

$$y_i = A_i F_i(l_{i1}, \ldots, l_{iM}, x_{i1}, \ldots, x_{iN}),$$

- A_i Hicks-neutral technology (Harrod-neutral as special case).
- x_{ij} intermediate inputs of good j used in the production of good i.
- l_{ik} labor of type k used by i.

Hulten's Theorem

Define $C(A_1, \ldots, A_N)$ to be competitive equilibrium aggregate consumption function interpreted as output.

Theorem 1.1 (Hulten)

Let λ_i denote industry i's sales as a share of output, then

$$\frac{\mathrm{d}\log C}{\mathrm{d}\log A_i} = \lambda_i.$$

Elasticity of Substitution

Definition 1.2

- For general CRS function $f(A_1, \ldots, A_N)$ define Morishima elasticity of substitution:

$$\frac{1}{\rho_{ij}} = -\frac{\mathrm{d}\log(MRS_{ij})}{\mathrm{d}\log(A_i/A_j)} = -\frac{\mathrm{d}\log(f_i/f_j)}{\mathrm{d}\log(A_i/A_j)}$$

  where $f_i = \partial f / \partial A_i$.

- For output function $C(A_1, \ldots, A_N)$, define *macro-elasticity of substitution*:

$$\frac{1}{\rho_{ij}} \equiv -\frac{\mathrm{d}\log(MRS_{ij})}{\mathrm{d}\log(A_i)} = -\frac{\mathrm{d}\log(C_i/C_j)}{\mathrm{d}\log(A_i)},$$

  where $C_i = \partial C / \partial A_i$.

- Note that $\frac{\mathrm{d}\log(\lambda_i/\lambda_j)}{\mathrm{d}\log A_i} = \frac{\mathrm{d}\log[(C_i A_i)/(C_j A_j)]}{\mathrm{d}\log A_i} = \frac{\mathrm{d}\log(C_i/C_j)}{\mathrm{d}\log A_i} + 1 = 1 - \frac{1}{\rho_{ij}}$.

Input-Output Multiplier

Definition 1.3

Define input-output mutliplier

$$\sum_{i=1}^{N} \frac{\mathrm{d}\log C}{\mathrm{d}\log A_i} = \sum_{i=1}^{N} \lambda_i = \xi .$$

- "Macro returns to scale": $\xi > 1$ implies reproducibility.
- ξ constant if and only if C homogenous of degree ξ.

Extending Hulten: Idiosyncratic Shocks

Theorem

$$\frac{\mathrm{d}^2 \log C}{\mathrm{d}(\log A_i)^2} = \frac{\lambda_i}{\xi} \sum_{j \neq i} \lambda_j \left(1 - \frac{1}{\rho_{ij}}\right) + \lambda_i \frac{\partial \log \xi}{\partial \log A_i}.$$

- General formula for second-order terms (nonlinearities) in terms of reduced-form macro-elasticities of substitution.
- Sales distribution not sufficient statistic.
- $\rho_{ij} = 1$, ξ constant: knife-edge case where effect disappears.

Extending Hulten: Common Shocks

Proposition 1.4

$$\frac{\mathrm{d}^2 \log C}{\mathrm{d}\log A_i \,\mathrm{d}\log A_j} = \frac{\lambda_i}{\xi} \sum_{k \neq j} \lambda_k \left(1 - \frac{1}{\rho_{jk}}\right) + \lambda_i \frac{\partial \log \xi}{\partial \log A_j} - \lambda_i \left(1 - \frac{1}{\rho_{ji}}\right) \quad (i \neq j)$$

- Shocks not additive.
- $\rho_{ij} = 1$, ξ constant: knife-edge case where effect disappears.

Macro Moments

Proposition 1.5

Suppose that $\log A_i$ *are subject to idiosyncratic shocks with variance* s_i^2*. Then we have the following formula for the mean of output:*

$$E(\log(C/\overline{C})) \approx \frac{1}{\xi}\sum_i \frac{s_i^2}{2\xi}\lambda_i \sum_{j\neq i}\lambda_j\left(1-\frac{1}{\rho_{ij}}\right)+\sum_i \frac{s_i^2}{2}\lambda_i \frac{\mathrm{d}\log\xi}{\mathrm{d}\log A_i}.$$

- See paper for:
  - more general mean formula for correlated shocks.
  - beyond mean, formulas for skewness and excess kurtosis.

Welfare Costs of Business Cycles

Proposition 1.6

Let $u : \mathbb{R} \to \mathbb{R}$ be a CRRA with parameter γ. Suppose TFP A has idiosyncratic shocks with variance s_k^2. Then the welfare costs of business cycles are given by:

$$\frac{\overline{C}[E(u(C)) - u(\overline{C})]}{u'(\overline{C})} \approx \underbrace{-\frac{1}{2}\gamma\sum_k^N \lambda_k^2 s_k^2}_{\textit{Consumption nonlinearities}} + \underbrace{\frac{\overline{C}}{2}\sum_k^N \frac{\partial^2 C}{\partial A_k^2} s_k^2}_{\textit{Production nonlinearities}},$$

where recall $\overline{C} = C(\overline{A})$.

- Nonlinearities in consumption: small cost in Lucas (1987).
- Nonlinearities in production: can be order of magnitude larger.

Mapping Micro Parameters to Macro Elasticities

Proposition 1.7

ρ_{ij} and $\mathrm{d}\log\xi / \mathrm{d}\log A$ can be solved for explicitly as a function of observable expenditure shares and micro elasticities of substitution.

- See paper for explicit characterization of reduced-form macro-elasticities in terms of micro primitives.

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Household

- Preferences given by consumption-bundle metric:

$$\frac{C}{\overline{C}} = \left(\sum_k b_k \left(\frac{c_k}{\overline{c}_k} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}.$$

- Consumer budget constraint:

$$\sum_k p_k c_k = \sum_k w L_k + \sum_k w_k l_k + \sum_k \pi_k.$$

Firms

- Industry k's production function given by

$$\frac{y_k}{\overline{y}_k} = A_k \left(a_k \left(\left(\frac{L_k}{\overline{L}_k} \right)^{\beta_k} \left(\frac{l_k}{\overline{l}_k} \right)^{1-\beta_k} \right)^{\frac{\theta_k - 1}{\theta_k}} + (1 - a_k) \left(\frac{X_k}{\overline{X}_k} \right)^{\frac{\theta_k - 1}{\theta_k}} \right)^{\frac{\theta_k - 1}{\theta_k}}$$

- X_k composite intermediate input given by

$$\frac{X_k}{\overline{X}_k} = \left(\sum_l \omega_{kl} x_{lk}^{\frac{\varepsilon_k - 1}{\varepsilon_k}} \right)^{\frac{\varepsilon_k - 1}{\varepsilon_k}},$$

where x_{kl} intermediate inputs from industry l used by industry k.

- L_k mobile generic labor and l_k fixed specific labor.

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Substitution and Reallocation

Proposition 2.1

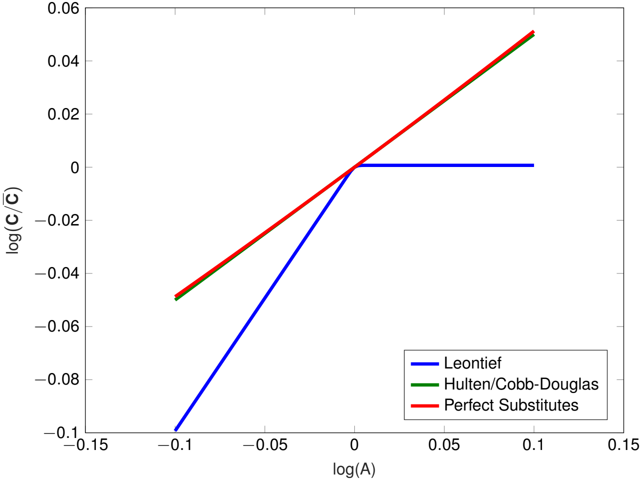
Suppose each good is produced using only labor. Assume uniform labor reallocation/returns to scale $\beta \in [0,1]$ for every k. Then

$$\rho_{ij} = \frac{\sigma(1-\beta)+\beta}{\sigma(1-\beta)+\beta+(1-\sigma)}, \quad \lambda_i = b_i, \quad \xi = 1, \quad \frac{\mathrm{d}\log\xi}{\mathrm{d}\log A_i} = 0.$$

- To build intuition, consider polar cases with $\beta = 1$ and $\beta = 0$.

Lesson #1: Micro-Elasticity of Substitution Matters

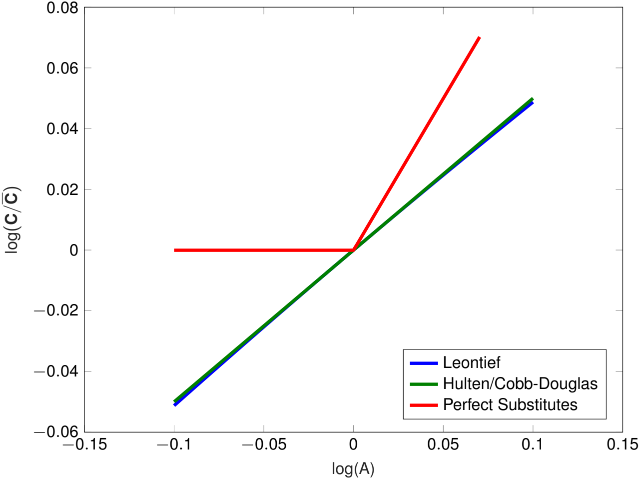
$$\beta = 0 \implies \rho = \sigma.$$



$$\frac{\mathrm{d}^2 \log C}{\mathrm{d} \log A_i^2} = b_i(1 - b_i)\left(1 - \frac{1}{\sigma}\right).$$

Lesson #2: Reallocation Matters

$$\beta = 1 \implies \rho = \frac{1}{2-\sigma}.$$



$$\frac{\mathrm{d}^2 \log C}{\mathrm{d} \log A_i^2} = b_i(1 - b_i)(\sigma - 1).$$

Network Irrelevance Result

Proposition 2.2

Let $\sigma = \theta_i = \varepsilon_i$, and consider Harrod-neutral (labor-augmenting) shocks. Then for any arbitrary network

$$\rho_{ij} = \rho, \quad \xi = 1, \quad \frac{\mathrm{d}\log \xi}{\mathrm{d}\log A_i} = 0,$$

where

$$\rho = \begin{cases} \sigma & \textit{if labor cannot be reallocated} \\ \frac{1}{2-\sigma} & \textit{if labor can be reallocated} \end{cases}.$$

$$\frac{\mathrm{d}^2 \log C}{\mathrm{d}\log A_i^2} = \lambda_i(1-\lambda_i)\left(1 - \frac{1}{\rho}\right).$$

- Extends Hulten network irrelevance to second-order.

Taking Stock

- General formula for second-order nonlinear effects of shocks in terms of macro-elasticities of substitution.
- Reduced-form macro-elasticities of substitution shaped by:
  - structural micro-elasticities of substitution.
  - factor reallocation and returns to scale.
- Network irrelevance if (1) uniform micro-elasticities, and (2) Harrod-neutral (labor-augmenting) shocks.
- Can break network irrelevance with (1) heterogenous production elasticities, and (2) Hicks-neutral (TFP) shocks.

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The role of ξ

- So far, $\xi = 1$, constant macro returns to scale.
- For most applications, $\xi > 1$: intermediate goods, capital, trade.

- In many applications, ξ restrictted to be constant: Gomme and Rupert (2007), Aghion and Howitt (2008), Jones (2011), Gabaix (2011), Acemoglu et al. (2012), Kim et al. (2013), Bartelme and Gorodnichenko (2015).

Variable ξ

- Assume

$$\frac{Y}{\overline{Y}} = A\left(\overline{a}\left(\frac{L}{\overline{L}}\right)^{\frac{\theta-1}{\theta}} + (1-\overline{a})\left(\frac{X}{\overline{X}}\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}},$$

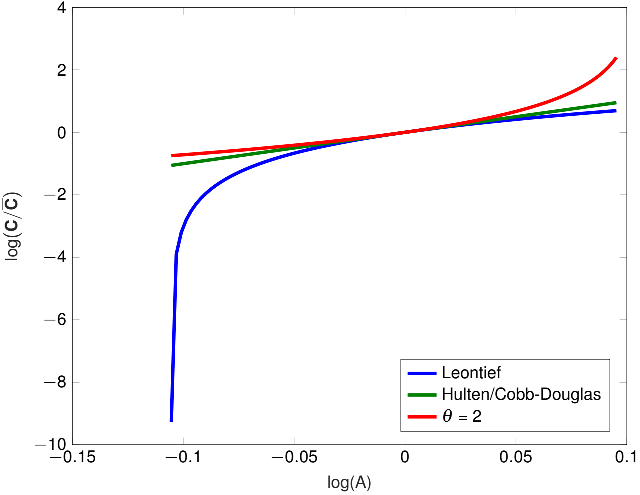
where

$$C + X = Y.$$

Proposition 2.3

$$\frac{\mathrm{d}^2 \log C}{\mathrm{d} \log A^2} = \left(\frac{1}{\overline{a}} - 1\right)(\theta - 1) = (\xi - 1)(\theta - 1).$$

Variable input-output multiplier



For this calibration, $\bar{a} = 0.1$.

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General Networks

Definition 2.4

The $N \times N$ *input-output* matrix Ω is the the matrix whose ijth element is equal to the steady-state value of

$$\Omega_{ij} = \frac{p_j x_{ij}}{p_i y_i}.$$

The *Leontief inverse* is

$$\Psi = (I - \Omega)^{-1}.$$

- Ψ_{ij} measures i's reliance on j.

Networks

Proposition 2.5

Assume $\beta_i = 1$ (CRS, full reallocation), and $\varepsilon_i = \theta_i$ for every i (w.l.o.g.), and Hicks-neuitral (TFP) shocks. Then

$$\frac{\mathrm{d}^2 \log C}{\mathrm{d} \log A_k^2} = (\sigma - 1) Var_b(\Psi_{(k)}) + \sum_j (\varepsilon_j - 1)\lambda_j Var_{\Omega^{(j)}}(\Psi_{(k)}).$$

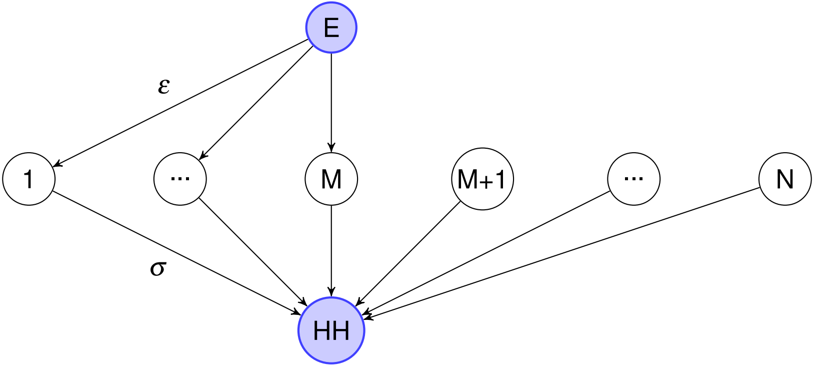
- Weighted variances of Leontief inverse:

  $Var_b(\Psi_{(k)}) = \sum_i b_i \Psi_{ik}^2 - \left(\sum_i b_i \Psi_{ik}\right)^2,$

  $Var_{\Omega^{(j)}}(\Psi_{(k)}) = \sum_i \Omega_{ji} \Psi_{ik}^2 - \left(\sum_i \Omega_{ji} \Psi_{ik}\right)^2.$

- Centrality measure mixing network and elasticities.
- Generalization to DRS and multiple factors.

Example: Universal Inputs



$$\frac{\mathrm{d}^2 \log C}{\mathrm{d} \log A_E^2} = (\sigma - 1)\lambda_E \left(\frac{N}{M} - 1 \right) \lambda_E + (\varepsilon - 1)\lambda_E \left(1 - \frac{N}{M}\lambda_E \right),$$

$$= \lambda_E(1 - \lambda_E)(\sigma - 1) + (\sigma - \varepsilon)\lambda_E \left(\frac{N}{M}\lambda_E - 1 \right).$$

Direction of Diffusion

Proposition 2.6

Consider two industries k and l that sell the same share to all other industries and the household $\omega_{ik} = \omega_{il}$ for each i and $b_k = b_l$. Then these industries are equivalent up to the second order:

$$\frac{\mathrm{d}\log C}{\mathrm{d}\log A_k} = \frac{\mathrm{d}\log C}{\mathrm{d}\log A_l},$$

and

$$\frac{\mathrm{d}^2\log C}{\mathrm{d}\log A_k^2} = \frac{\mathrm{d}^2\log C}{\mathrm{d}\log A_l^2}.$$

- Key: CRS and one factor.

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Simulation

- Set $\theta_j = \theta = 0.3, \varepsilon_i = \varepsilon \approx 0$, and $\sigma = 0.4$ drawing on Atalay (2016), Boehm et al. (2015), Barrot and Sauvagnat (2016), Comin et al. (2015).
- Set $(\sigma, \varepsilon, \theta)$ to match $\sum_i \overline{\lambda}_i \sigma_{\lambda_i} = 0.0197$.
- Impose no-movement in labor for benchmark (Acemoglu et al. (2016), Autor et al. (2016), Notowidigdo (2011)).
- Use the 88-sector US KLEMS annual input-output data from 1960-2005, with sector-level TFP data constructed using Jorgenson et al. (1987) methodology by Carvalho and Gabaix (2013).
- Set sectoral TFP shocks to be $\log \mathcal{N}(-\Sigma_{ii}/2, \Sigma_{ii})$, where Σ_{ii} is sample variance of $\Delta \log TFP$ for industry i.

Simulation Results

	Mean	Standard Deviation	Skewness
GDP Data	–	0.0238	-0.6190
TFP Data	–	0.0147	-0.2888
Benchmark	-0.0057	0.0117	-0.5229
Full reallocation	-0.0026	0.0110	-0.0745
Log Linear Hulten	-0.0010	0.0110	0.0000
Linear Hulten	0.0000	0.0110	0.0432
No Network, no reallocation	-0.0014	0.0053	-0.0420
No Network, full reallocation	0.0000	0.0053	0.0301
$(\theta, \sigma) = (0.1, 0.3)$	-0.0102	0.0138	-1.2864
High Volatility Benchmark	-0.0117	0.0180	-0.8821
High Volatility Hulten	0.0000	0.0155	0.0422

- Welfare costs of business cycles 0.57%, order of magnitude larger than those of 0.05% identified by Lucas (1987).

Histograms

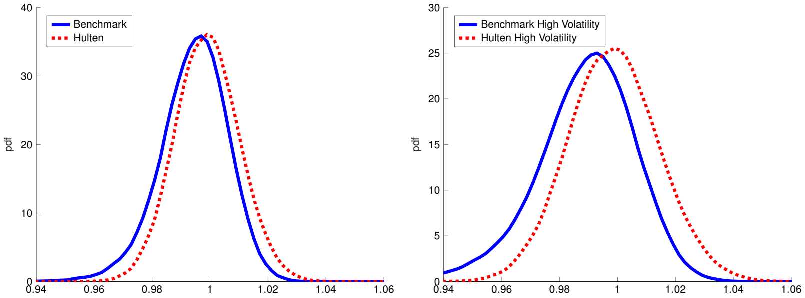
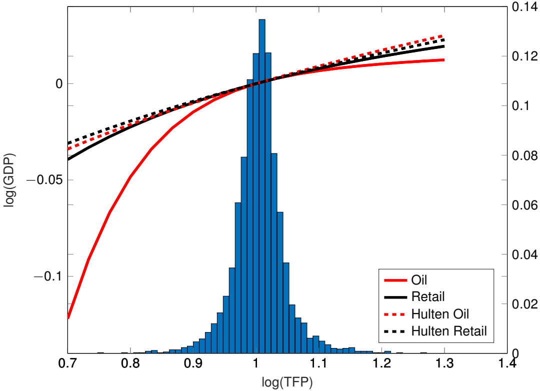


Figure: The left panel shows the distribution of *GDP* for the benchmark model and log-linearized model. The right panel shows these for shocks whose variance is twice as high.

- Excess kurtosis of 1 in benchmark model, increases with volatility.
- Endogenous and asymmetric fat tails (”rare disasters”).

Oil v. Retail



- Intuition: low micro-elasticity of substitution, universal input.
- Consistent with large asymmetric effects of oil shocks (Hamilton, 2003), even without frictions.

Reduced-form Impact of Oil Shocks

Proposition 3.1

Up to the second order in the vector Δ, we have

$$\log\left(C(A+\Delta)/C(A)\right) = \frac{1}{2}\left[\lambda(A+\Delta)+\lambda(A)\right]' \log(\Delta).$$

Reduced-form Impact of Oil Shocks

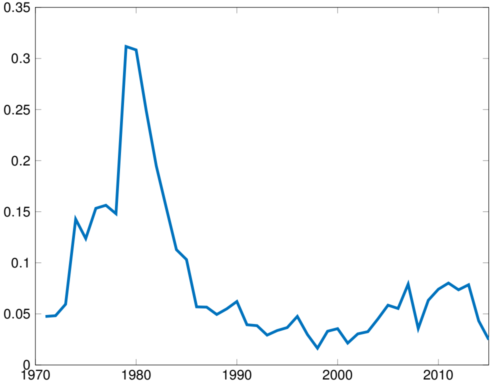


Figure: Global expenditures on crude oil as a fraction of world GDP.

- First-order effect: $5\% \times -13\% = 0.65\%$.
- Second-order effect: $\frac{1}{2}(5\% + 31\%) \times -13\% = 2.34\%$.

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- For empirically relevant cases, nonlinearities missed by first-order approximation are important.
- Second-order terms depend on macro-elasticities of substitution: macro-objects, not identified by micro-variation.
- Micro-elasticities of substitution, factor reallocation, micro-returns to scale, and networks play an important role in shaping these second order terms.
- Ongoing work to: allow for RBC channels (elastic labor supply, capital accumulation); dynamics (reallocation); frictions (markups/wedges); co-movement; macro-elasticities of substitution between factors; trade and gains from trade.

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